Structural Safety and Reliability, Shiraishi, Shinozuka & Wen (eds) © *1998 Balkema, Rotterdam, ISBN 90 5410 978 5*

Dynamic response variability of near-surface soil layer due to stochastic material properties

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ABSTRACT: The spatially dynamic response variability of near-surface soil layer during earthquake has been so far inferred mainly from the observation records of closely spaced seismic arrays. On the dynamic response variability, this study investigates the influence of laterally heterogeneous soil properties by means of first-order perturbation (FOP) and stochastic finite element (SFE) techniques. The theoretical FOP solution is derived first as a frequency-wave number power spectrum. The SFE analysis is then conducted much more effectively by proposing the assumption of the spatial ergodicity. Then, this study shows that the response variability due to the soil heterogeneity is significant over the first predominant frequency of a building site. By using the frequency-dependent correlation distance proposed as a simple measure characterizing the response variability, this study also demonstrates that, within the extent of the heterogeneity published, the FOP analysis is practically available for the site approximated well as a single soil layer.

1 INTRODUCTION

Local site effects have a significant influence on the various characters of earthquake ground motion, especially on the spatial response variabilities of near-surface soil layer. The stochastic analysis is in course of accounting for the significance of the local site effects pertaining the total wave propagation problem from seismic source to site of interest (Wu & Aki, 1988). Since we can not determine uncertain material properties, the stochastic analysis becomes more important when we understand the spatial statistics of the uncertainties related to such things as the seismic source, traveling path of seismic waves, and near-surface local site.

As far as the ground response at near-surface local site during earthquake is concerned, the spatial response variability so far has been inferred mainly from the observation records of closely spaced seismic arrays (Kataoka et al., 1990; Schneider et al., 1992; Nakamura, 1996). Some numerical analyses also have been made for the response variabilities. The stochastic analyses have been made for local site effects classified as the irregularities of free-surface and interface between soil layers (Harada, 1994), and the heterogeneity of soil properties (Frankel & Clayton, 1986; Harada & Fugasa, 1990; Fenton & Vanmarcke, 1991; Sato & Kawase, 1992). Deterministic analyses also emphasize the significance of surficial soil layer using the one dimensional multilayer model (Anderson et al., 1996).

As an extension of those stochastic analyses mentioned above, this study is also aimed at investigating the spatial response variability of surficial soil layer which has laterally heterogeneous soil properties, as shown in Fig. 1. While we have to consider the indeterministic soil properties by some way, it is common in the stochastic analyses to treat the indeterministic nature of structural systems as a stochasticity. Then, the response variability mentioned in this study is a

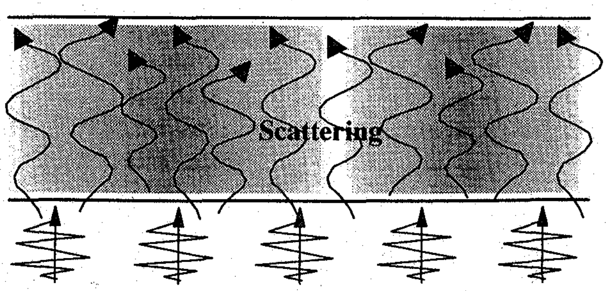

Fig. 1. Single soil layer model resting on rigid base.

Table 1. Parameters for laterally heterogeneous single soil layer model ($H = 30$ m).

Parameters	*Symbols*	*Values*
Density (t/m³)	ρ	1.8
Mean *P*-wave velocity (m/s)	c_{P0}	1530
Mean *S*-wave velocity (m/s)	c_{S0}	300
Poisson's ratio	ν	0.48
Correlation function	S_{xx}^{Δ}	Gaussian
Correlation length (m)	a	25 to 100
Coefficient of variation	σ_0	0.00 to 0.25
Intrinsic damping ratio	D	0.05

seismic version of the response variability since we adopt the same way that has been extensively studied in engineering mechanics field for structural systems with stochastic material properties (e.g., Nakagiri & Hisada, 1985).

This study analyzes the response variability of the ground by using an analytical approximation and a numerical discretization, which are first-order perturbation and stochastic finite element techniques, respectively. After each analysis is done for the seismic response variability, the comparison as well as discussion is made for those obtained from both analyses. Then, this study shows the practical availability of the perturbation analysis.

2 EVALUATION OF STOCHASTIC GROUND RESPONSE

This study analyzes the surficial ground motion that is the stochastic response of heterogeneous single soil layer induced by the coherent base motion. That is, this is an analysis to obtain the practical and hence simple solution of the ground response with laterally heterogeneous soil properties. Then, this study presents two efficient methodologies using a first-order perturbation method and a stochastic finite element method to obtain the power spectrum in frequency and wave number domains, since the space-time stochastic ground response can be represented by the frequency-wave number power spectrum. As well as the frequency-wave number response, the space-time response can be obtained in both analyses once the space-time incident wave is decided at the rigid base.

2.1 *First-order Perturbation Method*

A first-order perturbation technique is adopted so as to derive the analytically approximate solution of the problem involving material stochasticities of small lateral extent. This study considers a viscoelastic single soil layer overlying a flat rigid base as shown in Fig. 2. The single soil layer has the density of ρ, the complex elastic wave velocity of $c_J = c_J^0(1+iD)$ ($J = P, S$) where c_J^0 is the elastic *P*- and *S*-wave velocity, D is the linear hysteretic damping ratio, and $i = \sqrt{-1}$. The values of soil parameters used in this study are shown in Table 1. Since the constant thickness is supposed to be $H = 30$ m ($= z_1 - z_0$), the mean predominant frequency is 2.5Hz for the soil layer model. Such mean values of soil parameters are determined under the consideration of the Chiba site in Japan where the seismic array observation is performed since 1982 (Lu et al., 1990; Nakamura, 1996).

This study considers the *P-SV* waves subjected to a near-surface single soil layer. Following the Born-type approximation, the perturbation of elastic wave velocities is assigned in the single soil layer whose material properties vary laterally. The *P-SV* wave equation is solved theoretically by using an integral equation formulation (Kennett, 1972; Harada & Fugasa, 1990) on the Cartesian coordinate and corresponding displacement vector systems as shown in Fig. 2. Then, a closed-form analytic expression is obtained based on the propagator matrix method (Gilbert & Backus, 1966) for the laterally two dimensional single soil layer.

To obtain the perturbation solution, we utilize the condition that the vertical displacement is confined to zero to make the solution simple for practice. While the derivation of the perturbation solution is abbreviated, this paper shows a resultant solution of the frequency-wave number power spectrum describing the

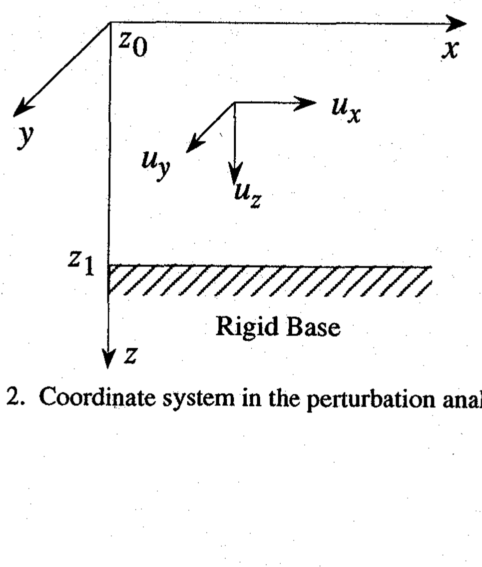

Fig. 2. Coordinate system in the perturbation analysis.

response variability due to laterally heterogeneous soil layer as mentioned above. The frequency-wave number power spectrum as functions of frequency, ω, in rad/sec and wave number vector, $\boldsymbol{\kappa}$, in rad/m is given by:

$$S^{S}_{xx}(\kappa,\omega)$$

$$=\left|K^{\delta}_{AP}\,\delta(\kappa-\kappa_0)-2K^{\Delta_S}_{AP}\,\overline{\Delta}_S(\kappa-\kappa_0)\right|^2 S^{B}_{xx}(\omega) \quad (1)$$

where δ = the Dirac's delta function and $\overline{\Delta}_S$ is the wave number spectrum describing the lateral heterogeneity of soil properties. Note that the heterogeneity of soil properties is not required to be isotropic in (1). $\boldsymbol{\kappa}_0$ and $S^{B}_{xx}(\omega)$ in (1) are the apparent wave number vector and the wave number power spectrum of the incident SV base motion, respectively. By using ν_{AP} and ν_{AP0} standing for vertical wave number corresponding to horizontal wave number, $\boldsymbol{\kappa}$ and $\boldsymbol{\kappa}_0$, respectively, K^{δ}_{AP} and $K^{\Delta_S}_{AP}$ in (1) are as follow:

$$K^{\delta}_{AP}=1\Big/\cos\left(\frac{c_P}{c_S}\nu_{AP}H\right) \quad (2a)$$

$$K^{\Delta_S}_{AP}=\frac{\nu_{AP0}{}^2+\kappa\kappa_0}{\nu_{AP}{}^2-\nu_{AP0}{}^2}\left\{K^{\delta}_{AP}-1\Big/\cos\left(\frac{c_P}{c_S}\nu_{AP0}H\right)\right\} \quad (2b)$$

especially when vertically incident wave is considered, i.e., $\boldsymbol{\kappa}_0=0$, K^{δ}_{AP} and $K^{\Delta_S}_{AP}$ at $\boldsymbol{\kappa}=0$ are given by:

$$K^{\delta}_{AP}=1\Big/\cos\left(\frac{\omega}{c_S}H\right) \quad (3a)$$

$$K^{\Delta_S}_{AP}=\frac{1}{2}\frac{\omega}{c_S}H\sin\left(\frac{\omega}{c_S}H\right)\Big/\cos^2\left(\frac{\omega}{c_S}H\right) \quad (3b)$$

In this case that $\boldsymbol{\kappa}_0=0$ and $\boldsymbol{\kappa}=0$, K^{δ}_{AP} and $K^{\Delta_S}_{AP}$ for confined P-SV wave correspond those for SH and SV wave. Note that the homogeneous and heterogeneous parts in (1) are coupled since $\overline{\Delta}_S$ is not zero at $\boldsymbol{\kappa}=\boldsymbol{\kappa}_0$ except special case. Considering this coupling is an amelioration of the solution proposed by Harada & Fugasa (1990).

If the heterogeneous part in (1) is neglected, this two dimensional perturbation solution, in fact, gives the same solution for one dimensional soil layer. While the perturbation solution expressed by (1) is derived for the confined P-SV wave, the perturbation solution for SV and SH waves is also derived in the same way (Harada & Fugasa, 1990). We suppose the case hereafter that S^{B}_{xx} is unity since this study is aimed at evaluating the response variability of single soil layer with stochastic material properties.

2.2 *Lateral Heterogeneous elastic wave velocity*

It is assumed in stochastic analyses that the spatial correlation at different locations in random field decreases with increase of the distance between two locations. To deal with the lateral heterogeneity of elastic wave velocities in single soil layer, this study supposes this assumption as well as many other researches (e.g., Frankel & Clayton, 1986; Fenton & Vanmarcke, 1991; Sato & Kawase, 1992) which have been done under this assumption to the correlation structure. Then, this study adopts the Gaussian correlation function, R^{Δ}_{xx}, and its corresponding wave number power spectrum, S^{Δ}_{xx}, as follow:

$$R^{\Delta}_{xx}(\xi)=\sigma_0{}^2\exp\left(-\frac{\xi^2}{a^2}\right) \quad (4a)$$

$$S^{\Delta}_{xx}(\kappa)=\sigma_0{}^2\frac{a}{2\sqrt{\pi}}\exp\left(-\frac{a^2\kappa^2}{4}\right) \quad (4b)$$

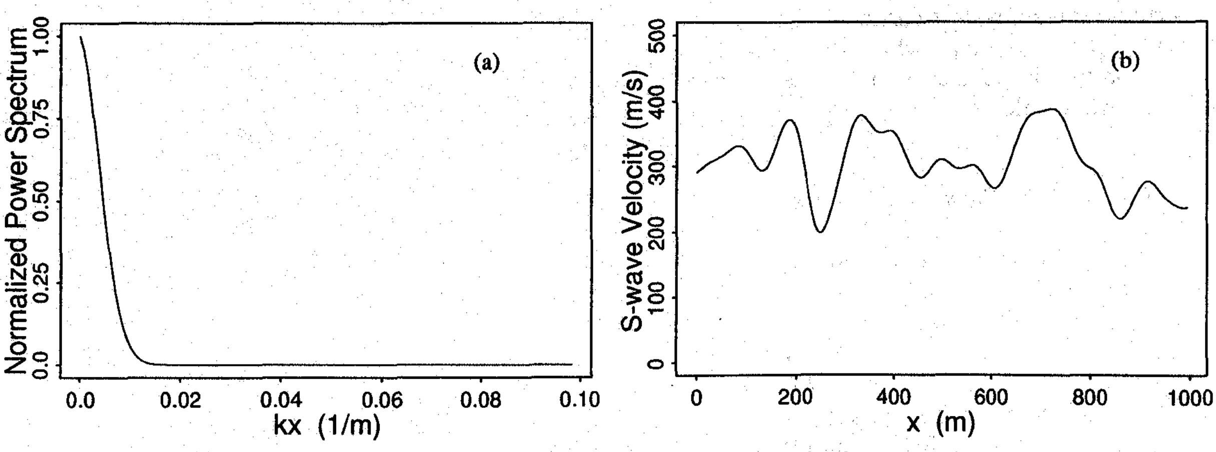

Fig. 3. (a) Wave number power spectrum describing laterally heterogeneous elastic wave velocity (correlation distance: a = 50 m; C.O.V.: σ_0= 15 %), and (b) the realization of the corresponding elastic wave velocity for S-wave with mean velocity of 300 m/s.

Table 2. Correlation distances of the lateral heterogeneities and sizes of finite element models.

Parameters		*FEM models* (1)	(2)	(3)	(4)
Correlation length (m)	a	25.0	50.0	75.0	100.0
Element width (m)	B_e	5.0	5.0	10.0	10.0
Total width (km)	B	1.0	1.0	2.0	2.0
Ratio of a and B_e	B_e/a	0.20	0.10	0.13	0.10
Ratio of a and B	B/a	40.0	20.0	26.7	20.0

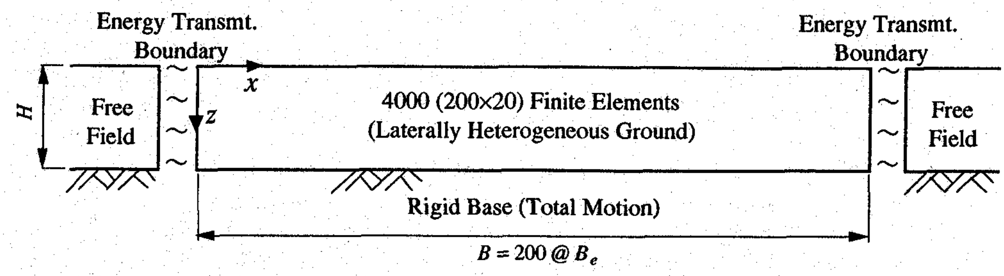

Fig. 4. Finite element model used in the response analysis of laterally heterogeneous soil layer.

where σ_0 and a indicate, respectively, the coefficient of variation (C.O.V.) and the correlation distance of laterally heterogeneous elastic wave velocity field. The extents of the C.O.V. and the correlation distance are determined based on the previous works (e.g., Fenton & Vanmarcke, 1991) as shown in Table 1. This Gaussian correlation function (4) is used in many other stochastic problems.

By using the Gaussian correlation function describing the stochastic elastic wave velocity, Fig. 3 (a) shows the example of wave number power spectrum in which the correlation distance, a , is 50 m and the C.O.V., σ_0, is 15 %, and Fig. 3 (b) shows the realization of the corresponding elastic wave velocity for S-wave with the mean velocity of 300 m/s. The correlation distance of 50 m and the C.O.V. of 15 % describing the stochasticity of the ground are used hereafter to show the response variability as a typical example. The perturbation analysis is carried out using the relation that $S_{xx}^{\Delta}(\kappa) = |\bar{\Delta}_S(\kappa)|^2$, while the stochastic finite element analysis is carried out using the corresponding elastic wave velocity.

2.3 *Stochastic Finite Element (SFE) Method*

In order to obtain the numerical solution of the same problem in the perturbation analysis and then to verify the perturbation solutions as shown above, a stochastic finite element (SFE) technique is adopted using a well-tested code, *super FLUSH* (KKEI, 1988). This finite element analysis code is used to compute the complex transfer functions at the surface of laterally heterogeneous single soil layer modeled as finite elements. This SFE analysis is conducted considering that the vertical displacement is confined to make the same condition as the perturbation analysis mentioned above.

As shown in Fig. 4, the finite element model consists of 200 by 20 plane strain elements laterally and vertically, respectively. The boundaries are modeled as the fixed boundary for the bottom and the energy transmitting boundary for the left and right sides. This boundary conditions give the same condition used in the perturbation analysis. Note that in this SFE analysis each soil column with same soil properties is also divided (30m = 10 @ 1 m + 10 @ 2 m) to obtain the better estimate of the surficial response. The free-field is also modeled using mean soil property of the finite elements, so that 2.5 Hz is the mean of the first predominant frequency at each location.

Table 2 shows the correlation distances of laterally heterogeneous soil layer and the sizes of finite element models in this SFE analysis. Note that since the soil property varies laterally, the lateral finite element size should be changed corresponding to the heterogeneity. To obtain the proper response of the heterogeneous ground model in this SFE analysis, we use the much smaller width of the finite element size (less than 0.20 times) and the much larger width of the finite element model (more than twenty times) compared to the correlation distance of the heterogeneity, as shown in Table 2. Therefore, the ground response in this study is considered to be statistically homogeneous.

Owing to the statistical homogeneity considered in space, we assume a spatial ergodicity which is the statistical technique showing that the spatial ensemble mean is independent of the spatial sampling. By using this spatial ergodicity assumption, the computa-

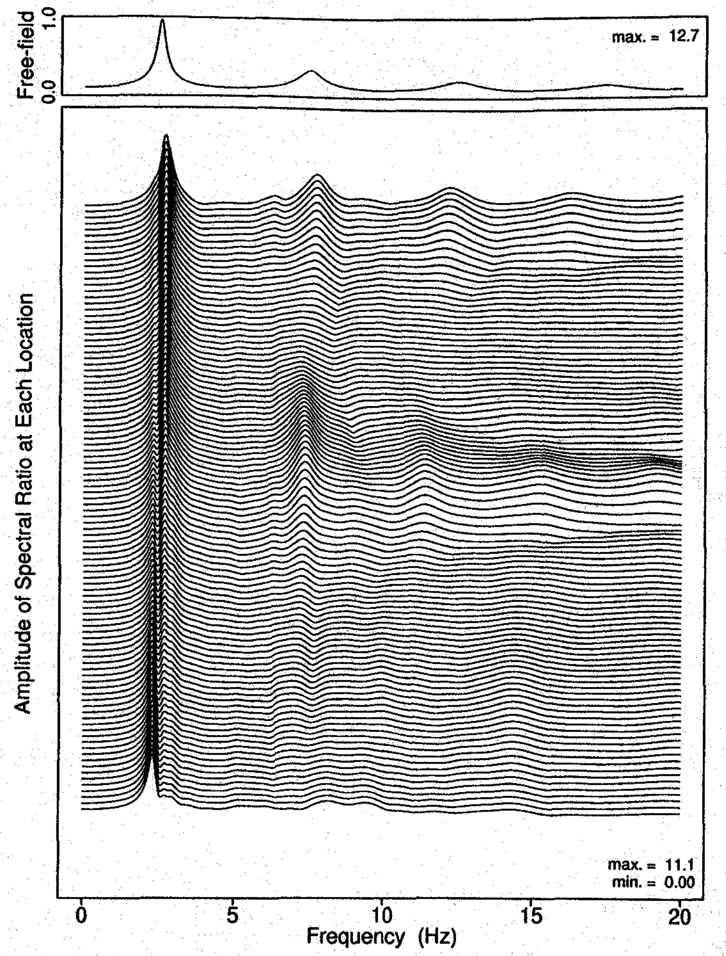

Fig. 5. Amplitude of complex transfer function estimated by the stochastic finite element analysis for laterally heterogeneous soil layer (a = 50 m, σ_0= 15 %; upper: for 1-D response, lower: for 2-D response).

tional efficacy is enhanced for the evaluation of the statistically homogeneous response variability, while the ordinary finite element analysis gives a deterministic result per computation once the material properties are specified in the finite element model.

The statistically homogeneous and stationary transfer function between ground surface and rigid base is obtained for the *P-SV* waves propagating in stochastic soil layer. Then, the amplitude of the complex transfer function is estimated for the laterally heterogeneous soil layer in which a = 50 m and σ_0 = 15 %, as shown in Fig. 5. The upper figure shows the response of the one dimensional soil column and the lower figure shows the response of the two dimensional soil layer ranging over 500 m at intervals of 5 m. As this study assumes that the base motion is unity, it is found in Fig. 5 that the ground response is affected over the first predominant frequency, especially at each predominant frequencies. Note that the result obtained from the assembly of one dimensional analysis should be different from this *P-SV* wave propagation analysis in which the responses of each soil column are affected each other.

3 RESPONSE VARIABILITY DUE TO STOCHASTIC GROUND MODELS

The stochastic ground response is computed by means of the perturbation analysis and the stochastic finite element analysis. As a result, it is found in both analyses that while the C.O.V. of the ground response shows the increasing trend with increase of frequency, the mean ground response shows good agreement each other and with that of the one dimensional analysis. Then, we pay attention to the response variability which is evaluated in terms of the coherence function and then the frequency-dependent correlation distance of the ground response which is pertaining to spatially varying earthquake ground motion.

3.1 *Coherence Function Describing Spatial Response Variability of Stochastic Ground*

The coherence functions are computed analytically in space and frequency domains to describe the spatial response variability of laterally heterogeneous

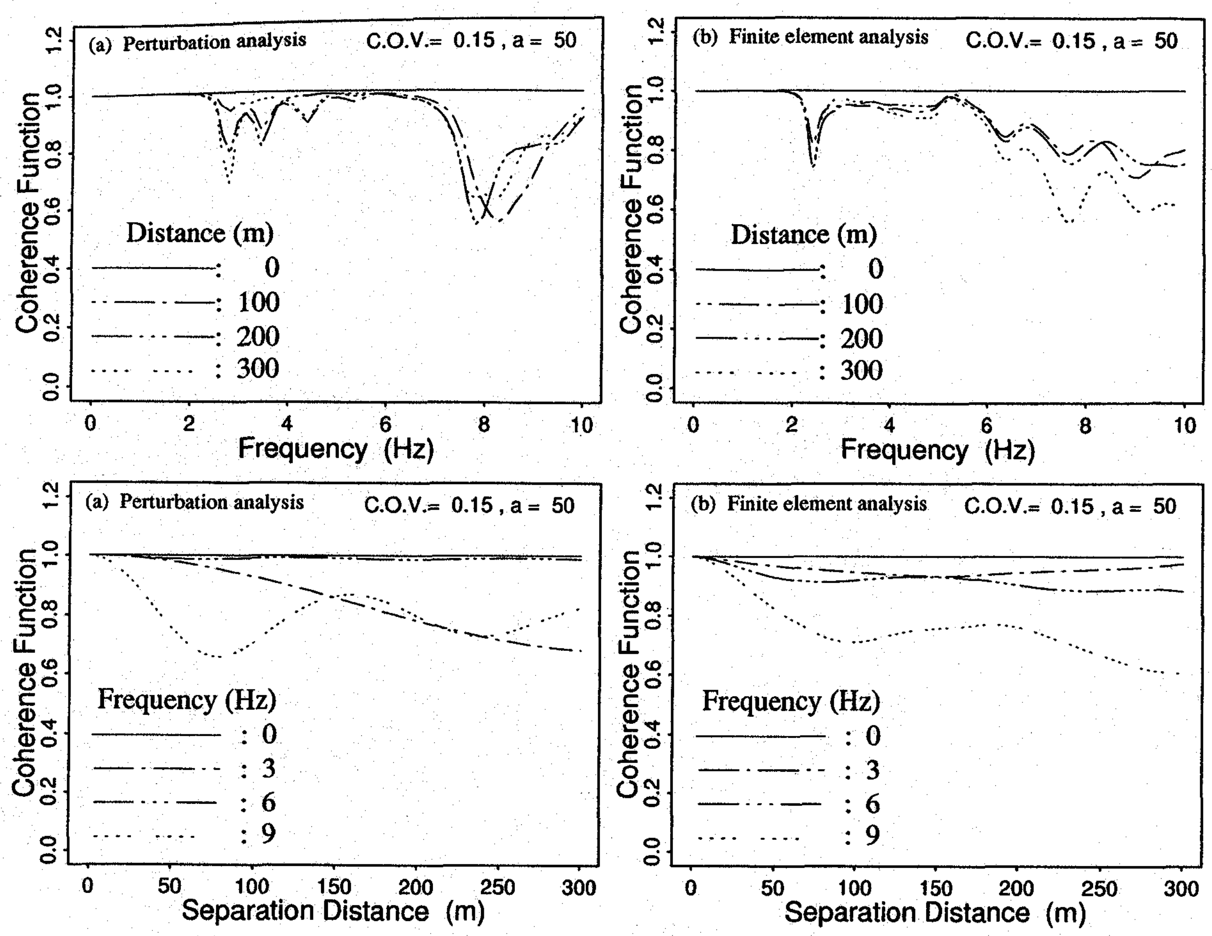

Fig. 6. Coherence function estimated from perturbation and finite element analyses for laterally heterogeneous soil laye ($a = 50$ m, $\sigma_0 = 15$ %; upper: frequency in Hz, lower: separation distance in meter).

ground models, since the frequency-wave number power spectrum is obtained from the first-order perturbation method and since the ground responses on surficial equidistant grid are obtained from the stochastic finite element method. Figure 6 shows the coherence functions representing the response variability of the ground model in which $a = 50$ m and $\sigma_0 = 15$ % as a typical example as mentioned above. The upper and lower figures indicate the coherence function with respect to frequency in Hz and separation distance in meter, respectively.

The coherence functions estimated from the both analytical and numerical analyses show the similar trend each other as shown in Fig. 6. It is found in this study that the coherence functions show the relatively monotonous decrease with increase of separation distance, while as frequency increases coherence function shows several decays especially around the dominant frequencies of the ground models. Furthermore, the coherence functions computed from both analyses are almost consistent with those obtained from the seismic array records which provides a useful information currently available (Kataoka et al., 1990; Lu et al., 1990; Nakamura, 1996). Then, the frequency-dependent correlation distance obtaine from the coherence function is adopted to evaluate th spatial response variability of laterally heterogeneou ground models.

3.2 *Frequency-dependent Correlation Distance o Stochastic Ground Response*

Suppose the coherence function decreases monotonously with respect to the separation distance. Then the seismic response variability is efficiently evaluated by the frequency-dependent correlation distanc of the ground response. For that reason, this stud adopts a Gaussian-type coherence function model a follows:

$$\gamma_{xx}(\xi, f) = \exp\left\{-\xi^2/q^2(f)\right\} \quad (5$$

where q indicates a frequency-dependent correlatio distance of the ground response estimated by the regression analysis. Note that the maximum separatio distance in this study is considered up to 300 m whicl is the same order in the analysis of the Chiba arra

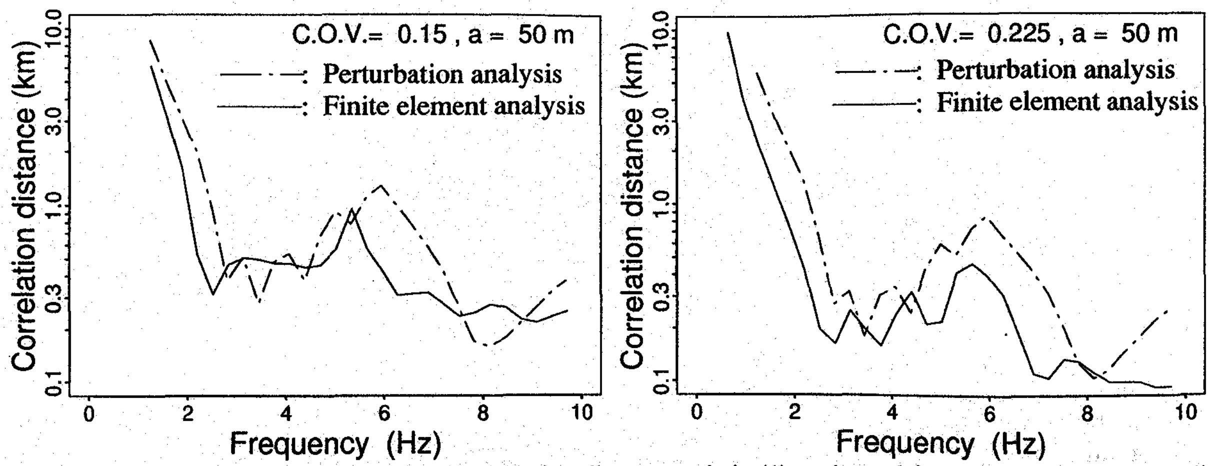

Fig. 7. Comparison of frequency-dependent correlation distances, q's in (1), estimated from perturbation (broken line) and finite element (solid line) analyses for laterally heterogeneous soil layer (left: case of $\sigma_0 = 15$ % and $a = 50$ m, right: case of $\sigma_0 = 22.5$ % and $a = 50$ m).

records (Nakamura, 1996).

It is demonstrated in this study that as the correlation distance (a) and C.O.V. (σ_0) of material properties increase, the spatial response variability shows the decreasing trend. Figure 7 shows the examples of the frequency-dependent correlation distance. The left and right show, respectively, the case that $\sigma_0 = 15$ % and $a = 50$ m and the case that $\sigma_0 = 22.5$ % and $a = 50$ m. Figure 7 also compares the frequency-dependent correlation distances q_P and q_F which are estimated from perturbation and stochastic finite element analyses, respectively.

To compare the both results, the L_2 norm difference between q_P and q_F is defined:

$$D_{PF}(\sigma_0; a) = \sqrt{\frac{\int_f \log^2\{q_P(f, \sigma_0)/q_F(f, \sigma_0)\}\, df}{\int_f \log^2\{q_F(f, \sigma_0)\}\, df}} \tag{6}$$

The L_2 norm difference indicates the difference of the results between perturbation and finite element analyses. That is, the results of both analyses become different from each other as the L_2 norm difference increases. Figure 8 shows the L_2 norm difference for laterally heterogeneous soil layer. The left and right figures indicate the cases of $a = 50$ m and the cases of $\sigma_0 = 15$ %, respectively. Note that, in both analyses, the frequency range of 2.5 to 10.0 Hz is used in the integration of above equations, since the ground responses of lower frequency components are coherent in this study, the frequency at least less than the predominant frequency of the ground, as can be seen in Figures 5, 6, and 7.

By comparing the results from perturbation analysis with those from stochastic finite element analysis, the largest L_2 norm difference is obtained in the case that $\sigma_0 = 22.5$ % and $a = 50$ m as shown in Fig. 8. It is found in Fig. 7 that even in the case of the largest L_2 norm difference, both results are allowable within the

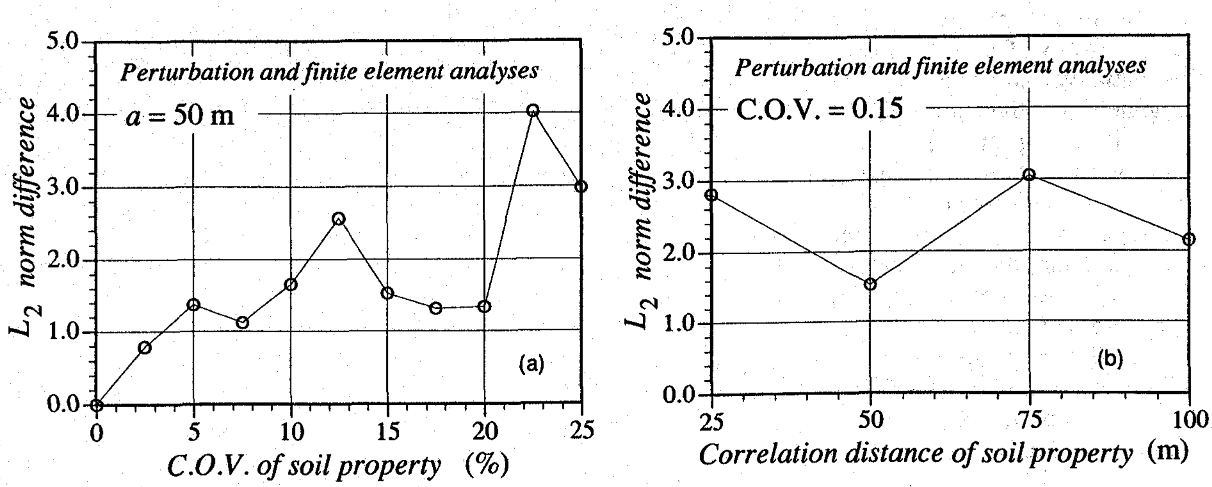

Fig. 8. L_2 norm difference between frequency-dependent correlation distances q_P and q_F, estimated from perturbation and finite element analyses, for laterally heterogeneous soil layer (left: $a = 50$ m, right: $\sigma_0 = 15$ %).

extent for frequency and separation distance of interest. Note again that the values of stochasticity for the material properties used in this study are carefully selected within the range properly estimated in previous works (e.g., Fenton & Vanmarcke, 1991).

4 CONCLUSIONS

Considering a stochastic approach to spatially dynamic response analysis, we have investigated the influence of the lateral heterogeneity of single soil layer on the response variability. The spatially dynamic response variability was examined by means of first-order perturbation and stochastic finite element techniques in this study. While the application treated is still simple, we hope that this study provides insight into the underlying mechanisms for further research to recognize the seismic response variabilities.

(1) The perturbation solution of frequency-wave number power spectrum has been derived theoretically using an integral equation formulation with the Born-type approximation. The perturbation analysis presented in this study is applicable to the laterally two-dimensional stochastic problem, while the effort at present has been concentrated on the laterally one-dimensional stochastic problem.

(2) By proposing the assumption of the spatial ergodicity, the stochastic finite element analysis has been conducted much more effectively, even though this analysis is still time consuming. Though used only for the single soil layer model, the finite element analysis is applicable for more complex heterogeneous ground model of interest.

(3) To verify the result of the perturbation analysis, we have proposed the simple measure which is the frequency-dependent correlation distance of the ground response. By using this simple measure, this analysis has demonstrated that the perturbation analysis is practically useful for the building site which is approximated well as a single soil layer. Both analyses can be used for the inversion analysis to estimate the heterogeneity of soil properties, if the major cause of the spatial variability is considered to be local soil heterogeneities at the site of interest.

(4) While the spatial response variability of the near-surface soil layer during earthquake has been so far inferred mainly from the observation records of closely spaced seismic arrays, this study also has demonstrated that the response variability due to the heterogeneous soil properties is significant over the first predominant frequency, especially at each predominant frequency. As a result, it is considered that the response variability of earthquake ground motion also may be manipulated considerably by the heterogeneity of the near-surface soil properties especially over the predominant frequency of the site.

ACKNOWLEDGMENT

This research was financially supported by the Grant-i Aid for Scientific Research for Japanese Junior Scienti: (No. 2827), the Ministry of Education, Science, and Cı ture.

REFERENCES

Anderson, J. G., Y. Lee, Y. Zeng, & S. Day 1996. Contı of Strong Motion by the Upper 30 Meters. *Bull. Seisı Soc. Am.* 86: 1749-1759.

Fenton, G. A. & E. H. Vanmarcke 1991. Liquefactiı Risk Assessment: 3-D Modeling. *Proc. 4th Int. Coı Seismic Zonation, EERI* II: 669-676.

Frankel, A. & R. W. Clayton 1986. Finite Differenı Simulations of Seismic Scattering: Implications for tl Propagation of Short-Period Seismic Waves in tl Crust and Models of Crustal Heterogeneity. *Geophys. Res.* 91: 6465-6489.

Gilbert, F. & G. E. Backus 1966. Propagator Matrices Elastic Wave and Vibration Problems. *Geophysiı* XXXI: 326-332.

Harada, T. & T. Fugasa 1990. Characteristics of Seism Responses of 3-Dimensional Ground with Stochast Soil Properties (in Japanese). *Memoirs of the Faculty Engineering, Miyazaki University* 36: 31-39.

Harada, T. 1994. A Stochastic SH Wave Model of Eartl quake Ground Motion. *J. Mech. Eng. Earthquake Enį JSCE* I-28: 43-50.

Kataoka, N., H. Morishita & A. Mita 1990. Spatial Vari: tion of Seismic Ground Motion at Lotung Soil-Struı ture Interaction Experiment Site. *Proc. 8th Japa Earthquake Eng. Symposium* 1: 607-612.

Kennett, B. L. N. 1972. Seismic Waves in Laterally Inhı mogeneous Media. *Geophys. J. R. astr. Soc.* 27: 30: 325.

KKEI 1988. *Super FLUSH*, Reference Manual. Koz Keikaku Eng. Inc.: Tokyo, Japan.

Lu, L., F. Yamazaki & T. Katayama 1990. Soil Amplific: tion Based on the Chiba Array Database. *Proc. 8th Jı pan Earthquake Eng. Symposium* 1: 511-516.

Nakagiri, S. & T. Hisada 1985. *Stochastic Finite Elemeı Method, An Introduction* (in Japanese). Baifu-kan: Tc kyo, Japan.

Nakamura, H. 1996. Depth-dependent Spatial Variatio of Ground Motion Based on Seismic Array Record: *Proc. 11th World Conf. Earthquake Eng.*: Paper Nc 731.

Sato, T. & H. Kawase 1992. Finite Element Simulation c Seismic Wave Propagation in Near-surface Randor Media. *Proc. Int. Symposium Effects of Surface Geoi ogy on Seismic Motion* I: 257-262.

Schneider, J. F., J. C. Stepp & N. A. Abrahamson 1992 The Spatial Variation of Earthquake Ground Motioı and Effects of Local Site Conditions. *Proc. 10th Worlı Conf. Earthquake Eng.* 2: 967-972.

Wu, R.-S. & K. Aki 1988. Introduction: Seismic Wavı Scattering in Three-dimensionally Heterogeneou Earth. *Pure Appl. Geophys.* 128: 1-6.