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RESPONSE OF RIGID BODY ASSEMBLIES TO DYNAMIC EXCITATION

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SUMMARY

Shaking table tests were conducted to investigate the response of rectangular wooden blocks and block assemblies of various sizes and slenderness to harmonic and earthquake base excitation. The shaking tests were followed by an analytical and a numerical study of response of single blocks and block assemblies. The analytical study was aimed at establishing criteria for the initiation of rocking and of overturning in response to harmonic base motion and consisted of solving numerically the differential equations of motion of a rigid block on a rigid foundation. The numerical study, in the course of which the response of both single blocks and block assemblies was examined, was implemented by means of the Distinct Element Method (DEM). Prior to the DE simulation of actual shaking tests, preliminary analyses were conducted to confirm numerical stability and to evaluate material and damping parameters. Comparing the recorded time histories with those given by the analytical study and the DE simulation, good agreement was found. The distinct element model in use appeared to follow the highly non-linear motion of rigid body assemblies faithfully to reality. On the basis of the results, provided that the necessary parameters are carefully estimated, the employed DE model can be regarded as an appropriate tool to simulate response of rigid body assemblies to dynamic base excitation.

INTRODUCTION

Residents of buildings are subjected to injuries from the displacement of surrounding objects (overturning of shelves, falling of overhead articles, etc.). Figure 1, taken in Kushiro City (Japan) after the 1993 Kushiro-Oki earthquake (M7·8) had struck, shows the interior of a building loaded with furniture. Many of these interior objects overturned in response to the intense ground shaking, inflicting potential danger to the inhabitants. The material loss due to the damage to the overturned and fallen objects was also considerable. To be able to estimate the stability of interior rigid body assemblies in the case of an earthquake, the response has to be followed from the onset of the excitation. With the analytical computation of motion of rigid body assemblies exposed to dynamic excitation being highly non-linear, and, as such, extremely complex to derive analytically, explicit numerical approaches that can handle a large number of arbitrarily shaped bodies and can regard various kinds of boundary conditions may be much more convenient to apply.

The problem of overturning of rigid bodies due to ground shaking has attracted many researchers' attention. Research in this field dates from the 19th century. In Japan, before the development and setting up of modern earthquake observation facilities, horizontal peak ground accelerations and the directions of shocks were attempted to be anticipated on the basis of overturning of columns of various sizes and shapes. The first publications on this subject were those of Milne[1] and Perry.[2] Substituting the breadth (b) and height (h) of those that overturned into West's formula,[3] the horizontal peak ground acceleration was estimated as

$$a > \frac{b}{h} g \tag{1}$$

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Figure 1. Interior with overturned furniture after the Kushiro-Oki earthquake of 1993

where g is the gravitational acceleration. A comprehensive paper on the method of estimating peak ground acceleration from fallen tombstones was published by Omote *et al.*[4] They have investigated overturned gravestones at ten graveyards in the epicentral area of the Ohita earthquake of 1975. Analogously to Milne's, they anticipated the maximum horizontal acceleration in the epicentral area by West's formula. This method has a number of shortcomings and thus results from it are often controversial. Looking at tombstones of identical breadth and height measurements at a graveyard, some may overturn while others may remain standing depending upon their orientations and the conditions of the soil in which their foundations are embedded. Surveys of damage caused by strong ground shaking, for example in the Middle-East or in Chile (1960), where tall and slender monuments survived while seemingly stable ones in their surroundings were severely damaged, suggest that instead of representing the ground motion by an instantneous static force, it is necessary to consider the characteristics of the ground motion and the non-linearity of the response of the rigid block.

The mathematical model used in the analytical study (in Method 1) in this paper has been adapted by many researchers studying the response of rigid bodies to seismic excitation. The first one to apply this method was Housner.[5] He derived the minimum horizontal acceleration of a single pulse necessary to overturn a rigid rectangular block. This genuine study was followed by a series of experimental and numerical studies. Focusing on the rocking motion of a rigid block on a rigid base subjected to horizontal and vertical ground motion, a numerical procedure and a computer program were developed by Yim *et al.*[6] They examined overturning by single pulse excitations and by natural earthquake motions. They also studied the influence of system parameters and ground motion properties on the response; firstly, based on computed response characteristics and also probabilistically, with respect to the variations of the coefficient of restitution, the intensity of the input ground motion, the b/h ratio and the size of the block. Finally, they proposed a probabilistic estimation of the intensity of ground motion. Ishiyama[7,8] also studied the response of a single block on a rigid foundation to harmonic and earthquake ground motion. The equations of motion of a rocking rigid block, considering sliding and bouncing, were formulated and solved numerically. New criteria of rocking and overturning with respect to peak ground acceleration and velocity were proposed. Tso and Wong[9] studied the conditions of steady-state rocking response of rigid blocks on rigid foundations. Waveforms and amplitudes computed from the equation of motion of a rocking block were found to be in good agreement with those resulting from shaking table tests. Their work provides further insight into the complex behaviour of rocking blocks.

Dynamic behaviour of a rocking rigid block supported by flexible foundations which allow for uplift was analysed by Psycharis and Jennings.[10] In their work, two foundation models were compared, the Winkler foundation and a simpler two-spring foundation. They showed that it is permissible to model a continuous flexible foundation by means of a discrete, two-spring model when analysing the dynamic behaviour of a rigid block. The two-spring model they used to represent a foundation is very similar to the contact model of the distinct element program employed by the present authors. Very comprehensive studies of the governing equations of a rocking block, with particular emphasis on system parameters and initial conditions, and of the motion of a rigid block under harmonic forcing were made by Hogan.[11,12]

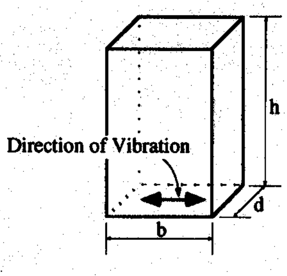
Recent studies extend the problem of rocking of a rigid block on a rigid foundation to the examination of more complicated body assemblies. One of these is an experimental study[13] on the behaviour of sliced solid body assemblies subjected to horizontal base motion. However, the experiments were not followed by simulation. Psycharis[14] analysed the response of a two-block assembly in which a rectangular block is placed on top of another. The analytic formulation of the solution of this highly non-linear problem proved to be very complicated. Governing equations for all of the possible vibration modes were derived and the rate of energy dissipation was also evaluated for each mode.

Since the existing studies and simulation models are limited to represent either one block or two-block assemblies with simple boundary conditions, the authors seek an approach that is applicable to rather general models and boundary conditions. The Distinct Element Method (DEM), originally proposed by Cundall,[15] was selected for the purpose of numerical simulation. A system of rigid blocks in equilibrium is highly indeterminate, that is, there are a large number of particular force distributions that would satisfy equilibrium. Whatever force distribution exists at a given time is largely a function of the previous history of the system. In view of this, it seems to be safe to use an explicit method, which advances in time by very small steps, opposed to an implicit method, which, although it can take much larger time steps, may step over some important, irreversible effects. For the DEM, each iteration represents a real time step by using the correct law of motion. In this study, the DEM is highlighted as a tool to simulate the dynamic behaviour of rigid block assemblies under base excitation.

SHAKING TABLE EXPERIMENTS

A series of shaking table tests were conducted in the Chiba Experimental Station of The University of Tokyo using a two-directional (horizontal and vertical) shaking table. Parallelepipeds, listed in Table I, glued from thin wooden plates were used for specimens. The size of the blocks was determined such that small disturbances like the attachment of pickups and cables would not affect their behaviour considerably. The solid, composite material the blocks were made of provides that they can be regarded as rigid bodies. In order to increase the friction coefficient between the wooden plate on the shake table and the specimens to a degree that sliding would not occur, that is, that the friction coefficient be greater than the b/h ratio of the block,[16] abrasive paper was attached on the lower base surfaces of the specimens. To monitor the motion of the specimens, two accelerometers for measuring horizontal and vertical acceleration were attached to each

Table 1. Table of specimens



Block	b [cm]	h [cm]	d [cm]	k	b/h
Block 1	30	100	50	2.0	0.3
Block 2	22.5	75	37.5	1.5	0.3
Block 3	15	50	25	1.0	0.3
Block 4	7.5	25	12.5	0.5	0.3
Block 5	25	50	25	-	0.5
Block 6	20	50	25	-	0.4
Block 7	10	50	25	-	0.2

block. The measured acceleration corresponding to each one of the two pickups used is horizontal or vertical only until the angle of rotation becomes non-zero, that is, the block is set into rocking. From that moment, it may be very difficult to compute purely horizontal or vertical acceleration from the records as it would require a continuous changing of the orientation of the co-ordinate system of the records combined with numerical integration from acceleration to displacement in order to establish the new angle of the co-ordinate system. As a consequence of this condition, it has not been possible to compare directly displacement time histories from the shaking tests, the analytical study and the DE simulation; however, for the procedure of fitting the displacement time histories of free vibration the computation of displacement from the channel records was performed. Thus the authors had to resort to establishing the moment of onset of rocking and the moment of overturning from vertical acceleration records (Figures 2 and 3). These data could then be compared with time histories of angle of rotation from the analytic study and from the DE simulation. To facilitate direct comparison of motion, a system for detecting displacements of corners of blocks is desired.

The shaking table tests can be divided into two groups: tests on single blocks and tests on block assemblies.

Single blocks

The aim of the tests on single blocks was to examine the effects of block size and slenderness on the initiation of rocking. Two groups of blocks, one of varied heights and identical b/h ratio and the other one of four blocks of identical height but of varied slenderness, were tested. All of the shaking table tests on single blocks were conducted using base motions of frequencies from 1 to 5 Hz in 0·5 Hz steps.

Block assemblies

Tests on different combinations of blocks were conducted using harmonic and natural earthquake excitations. Columns, composed of two or three blocks ($h = 50$ cm, $b/h = 0{\cdot}3$), were tested under different boundary conditions in order to confirm how accurately the numerical simulation follows the highly non-linear response of complicated assemblies. The simplest assembly tested was a two-block column in which a block was placed on top of another. It was similar to one, the behaviour of which was studied

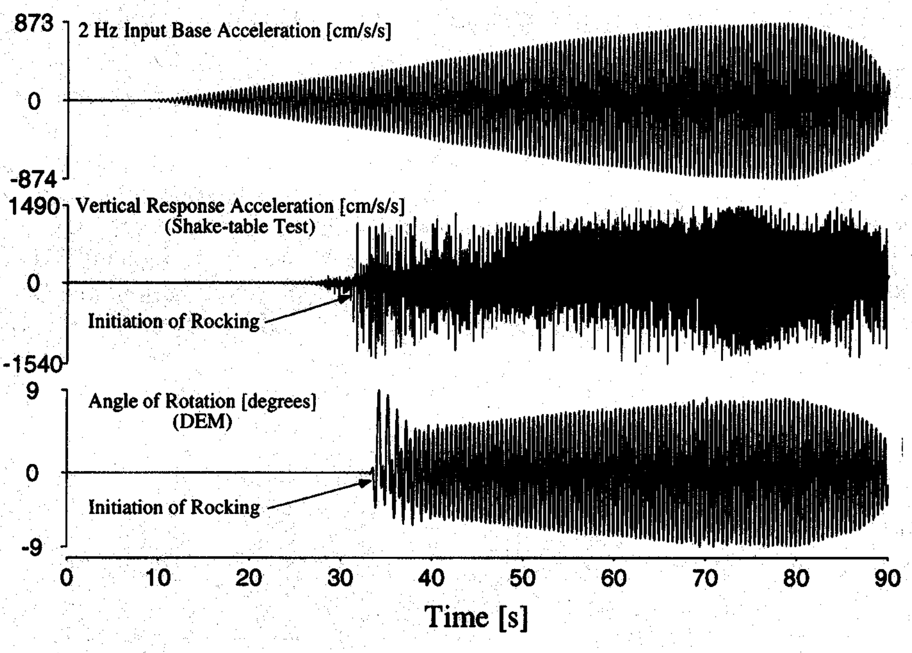


Figure 2. Detection of the initiation of rocking for Block 2 (shake-table tests—DEM)

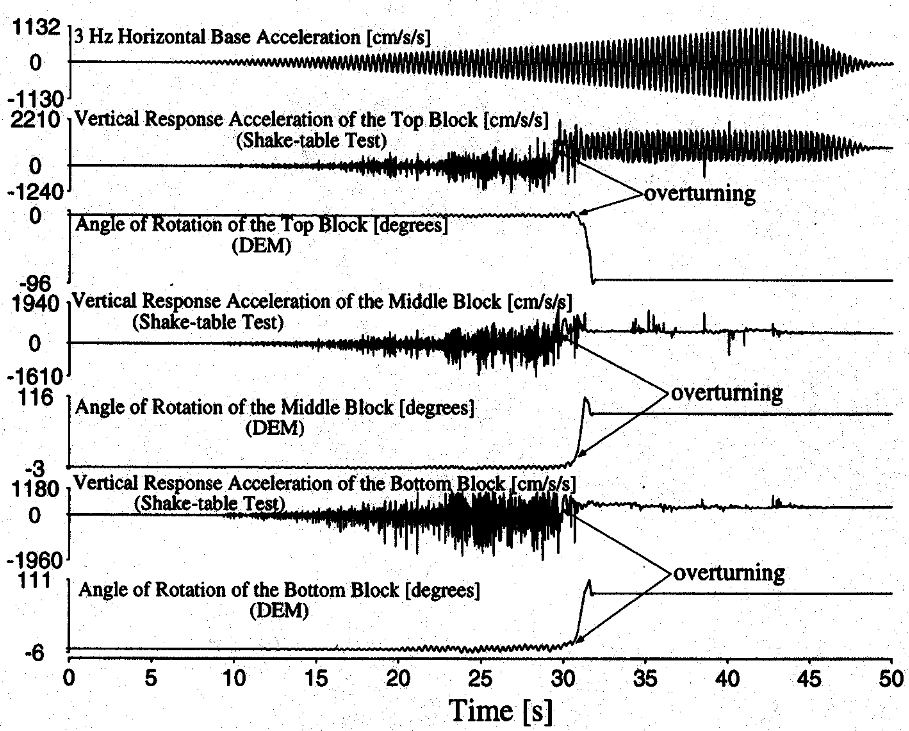


Figure 3. Detection of the moment of overturning for the three-block column (shake-table tests—DEM)

analytically by Psycharis.[14] To make the response more non-linear and therefore more complicated, the above column was also tested set next to a wall that was firmly attached to the shaking table. The example thus created resembles that of bookshelves and cupboards that respond to ground motion in interaction with a wall. The most complex assembly that was tested was a freely standing column of three blocks.

ANALYTICAL RESPONSE ANALYSIS FOR SINGLE BLOCKS

The response of single blocks to harmonic horizontal base motion was estimated by two different methods. The input base excitations used in these analyses were recorded in the shaking table tests. Since there were only a limited number of cases of overturning in the shaking table tests the input motion records were scaled up to allow for larger base acceleration values and consequently for overturning at all used frequencies. For a later comparison with results from the DE simulation, for which input motion amplitudes were also scaled up, criteria of the initiation of rocking were computed for two sets of single blocks: blocks of different sizes (Blocks 1, 2, 3 and 4) and blocks of varied slenderness (Blocks 5, 6, 3 and 7). To compare criteria of overturning from the analytic approaches and from the DE simulation, three blocks (Blocks 1, 3 and 7) were chosen.

Method 1. The first method is a simple analytical approach in which the governing equations of a rocking rigid, rectangular block on a rigid floor (Figure 4) are solved numerically. The equations used were originally derived by Yim *et al.*[6] and are written, depending upon whether the block is rocking about O or O′, as

$$I_0\ddot{\Theta} + WR\left(1 + \frac{a_g^y(t)}{g}\right)(\Theta_c - \Theta) = -\frac{W}{g}Ra_g^x(t) \tag{2}$$

$$I_0\ddot{\Theta} - WR\left(1 + \frac{a_g^y(t)}{g}\right)(\Theta_c + \Theta) = -\frac{W}{g}Ra_g^x(t) \tag{3}$$

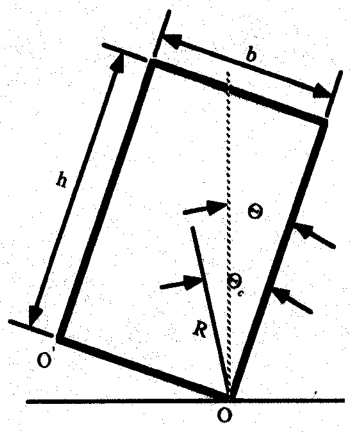


Figure 4. Rocking block

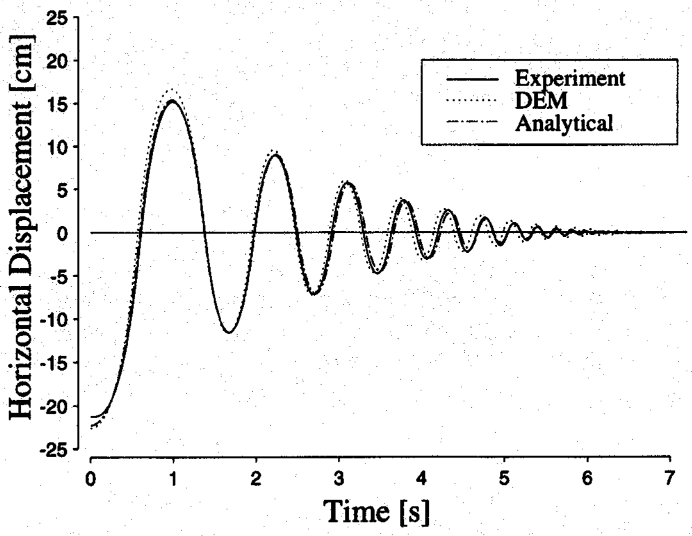


Figure 5. Time histories of free vibration of a rigid block (Block 1)

where $a_g^x(t)$ and $a_g^y(t)$ are base accelerations in the horizontal and in the vertical direction respectively, I_0 is the mass moment of inertia of the block around its base edges, W is the weight of the block, g is the acceleration of gravity and R is the half of the diagonal of the b/h-face of the block. Θ and Θ_c are illustrated in Figure 4. The coefficient of restitution, established by fitting the computed response-curve of a block in free vibration to that observed in the experiment (Figure 5), is given as

$$e = \dot{\Theta}_2/\dot{\Theta}_1 \tag{4}$$

where $\dot{\Theta}_2$ and $\dot{\Theta}_1$ are the angular velocity values after and before an impact with the base, respectively.

Method 2. Due to extremely large discrepancies between criteria of overturning from Method 1 and from the DE simulation, a more sophisticated analytical method was sought. A primary concern about the second method-to-be was that it allows for non-linear effects, such as sliding and occasional separation from the floor, that were not taken into consideration in Method 1.

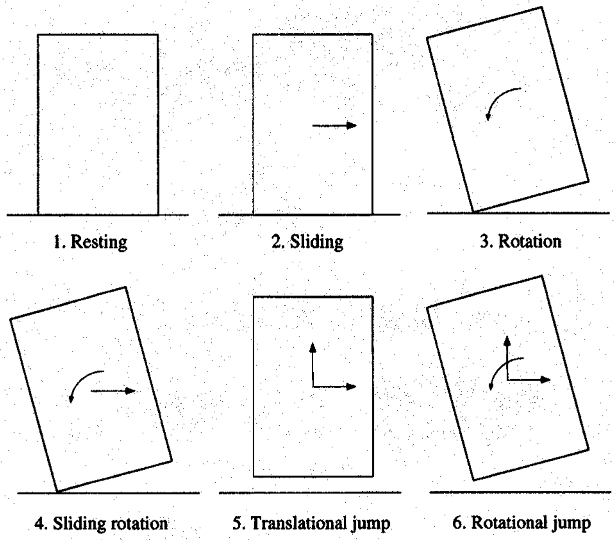


Figure 6. Possible modes of response in Method 2

The authors selected the method developed by Ishiyama[8] to compute the response of rigid bodies to earthquakes. This approach regards six different possible modes of response (Figure 6). After every cycle of computation, a subsequent type of motion is selected on the basis of the response hitherto. Then the corresponding set of equations of motion is solved; the next type of motion is chosen and the cycle repeats itself. As for realization of energy dissipation, two coefficients of restitution, one normal and one tangential, are introduced. Distinction is made between centric (e.g. in the case of translational jump) and eccentric (e.g. in the case of rotation) impacts between the body and the floor and the corresponding equations of energy dissipation are derived for each of the six modes.

In order to be able to understand why Method 2 is superior to Method 1 one has to have a very close look at the response. At high base accelerations it is very difficult to distinguish between rotational jump and sliding rotation. Since rotational jump ensues only for a few instants and the uplift is very small, it may seem as though the block were sliding. What really happens is that due to the uplift there will be a relative displacement between the floor and the block (and friction has no meaning any more). At the moment the contact is restored there are horizontal and vertical impacts from the floor that interfere with the response of the block hitherto. These are such non-linear fragments of response that are not allowed for in Method 1 but are essential to the whole of the response. When one looks at the animation from the DE simulation and follows the fashion of response at high base accelerations then it is undoubtedly proven that the described phenomena do take place when coming into contact with the floor. It has to be emphasized again at this point that the response one encounters is highly non-linear and passing over even a seemingly minor nuance of it one may come to an essentially different final state. This is also a reason why the value of the time increment for the DE simulation bears grave importance and has to be selected very carefully.

ABOUT THE DISTINCT ELEMENT METHOD (DEM)

The method of calculation is essentially similar to that described by Cundall[17] in 1971.

Basic assumptions. In the two-dimensional DE (Distinct Element) program used for the computation of behaviour of assemblies of rigid blocks, there is no restriction of block shapes and no limitation to the magnitudes of translational and rotational displacements. In this method the approximation that all

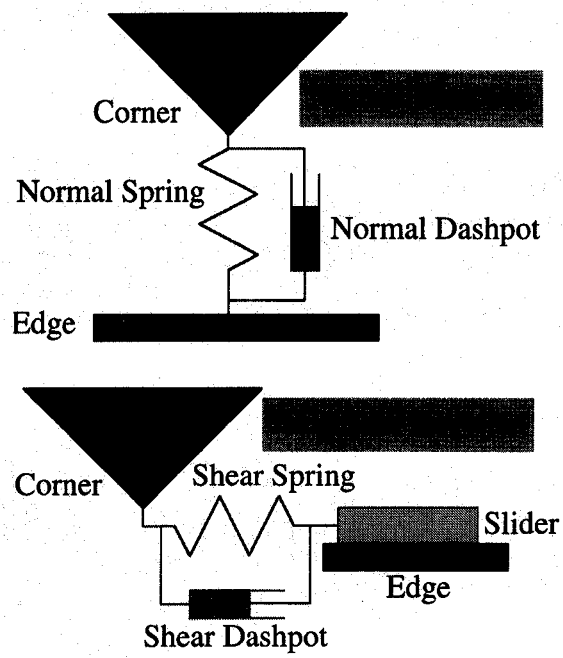


Figure 7. The distinct element model

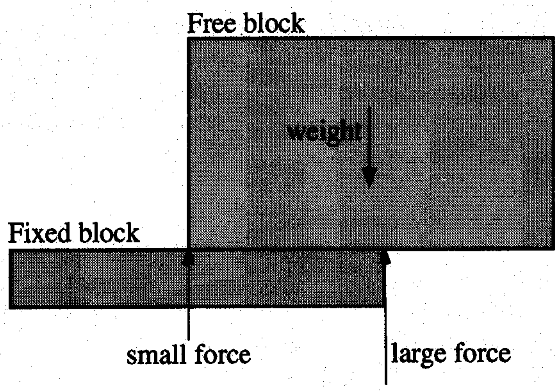


Figure 8. Adjustment of the contact forces at an edge/corner contact according to Cundall's DE model

deformations occur at the surfaces of blocks is made. Also, it is assumed that the deformations are localized at each contact between a corner and an edge; forces arise only at edge/corner contacts (Figure 7). The equilibrium of two edges that are in contact is established by the fact that the forces at the two corners adjust themselves to be in the ratio required for equilibrium (Figure 8).

Computation of forces from displacements. Forces arise due to deformation. In the DE program used the force/displacement laws are incremental. That means that a change in displacement results in a change in force which is added to the existing force stored for the contact. Within one time step the incremental normal and shear displacements for a given contact are evaluated from the incremental X, Y and rotational displacements for the two blocks concerned. The new normal and shear forces are then calculated from the old forces using the incremental forces calculated from force/displacement equations. These contact forces are then transformed into equivalent X and Y forces and moments, and added to the X and Y forces and moment acting on each block.

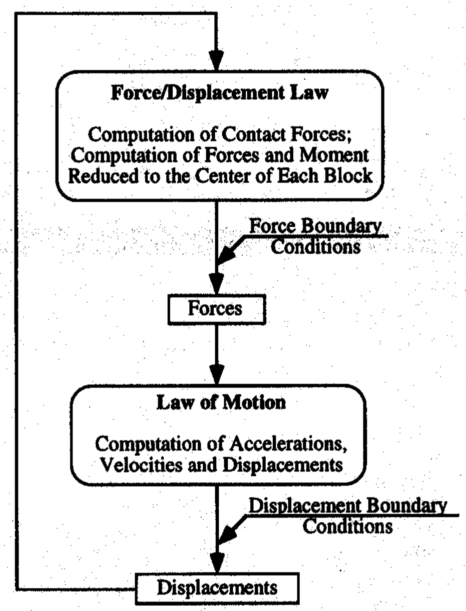


Figure 9. The flow of the DE computation

Law of motion. From the forces and moment for every block, accelerations are computed in X, Y and rotational directions. The accelerations are then further integrated into velocities and displacements. Having computed the new displacements we are back at the beginning of the computational cycle (Figure 9) and are to compute the new contact forces.

Contact updating and contact judgement. Contact updating, that is, checking for new contacts, is not done in each cycle of the computation as it would make the computation much more time consuming. Updating of the contacts is done when the sum of the displacements of all of the elements has exceeded a certain value. Owing to the non-linear characteristics of the phenomena we use the DEM to give reference about the constant for contact updating is of great importance. It was chosen as small as possible to allow for running within a still tolerable CPU time.

The parameters for contact judgement are another factor that add to the complexity of the influence of the boundary conditions when one uses the DEM to simulate the rocking response of rigid blocks. Reality, when regarding 'rigid' bodies, bodies of high material stiffness, takes care of the matter of establishing and abolishing the contact between two bodies rather unambiguously. In the DE simulation contact judgement is a very important set of decisions to be made. There are several parameters that should be incorporated, yet there has not been a comprehensive examination as to the quantitative influence of all those. It has to be decided how far the surface (edge) of a block may be from a corner of another to have to be considered to be 'in contact' already. In reality, when there are rigid blocks in question, the value of this distance is very near zero. In the DE simulation this value is influenced by the velocity at which the elements travel, by the time increment that will tell how far they travel during one time step at that velocity and also the material stiffness and damping parameters that will limit the intrusion of a corner into an edge in contact. If one could afford to use an infinitely small time increment the problem would be solved as easily as it is in reality. The establishment of a problem-dependent function that takes these quantities into account would require specific laboratory testing and the authors had to resort to the use of reasonably estimated constants for contact judgement and contact updating.

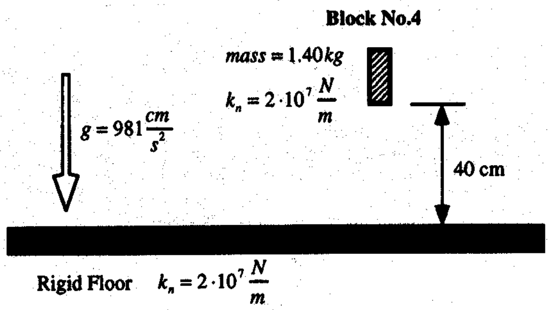


Figure 10. Numerical stability test through the undamped oscillation of a rebounding rigid block under gravitation

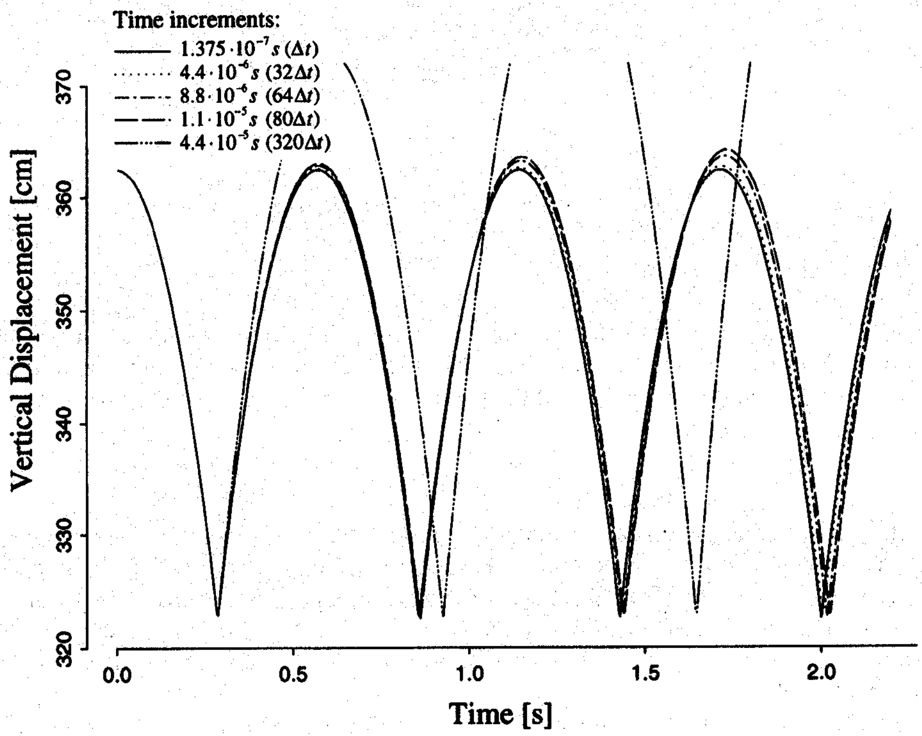


Figure 11. Vertical displacements of the oscillating rigid block for different time increment values

DISTINCT ELEMENT SIMULATION

Preliminary analyses

Numerical stability. Since, physically, rigid blocks communicate with each other at a speed somewhat lower than the speed of sound in the material, the numerical method must calculate faster than the propagation of the stress wave. It follows that the time step in explicit methods must be smaller than some critical time increment, which is a function of the mass and stiffness of the system components. To select an appropriate time step, a test for numerical stability was performed (Figure 10). In this test computed time histories of the response of a block falling under gravitation on a rigid base, using various values of the time increment (Figure 11), were compared and a properly small time step that would ensure numerically stable computation was selected. For one degree of freedom, the equations of motion are

$$F_{\text{spring}} = -kx \tag{5}$$

$$F_{\text{mass}} = m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \tag{6}$$

in which k is the spring constant, x is the elongation of the spring, m is the mass and t is time. These equations can be solved[17] giving stable solutions for time increment values derived from

$$\Delta t < 2\sqrt{\frac{m}{k}} \tag{7}$$

where Δt is the time increment. However, k is not the contact stiffness from equation (5), but is the stiffness the block encounters upon contact with other blocks. Every surrounding block makes the apparent stiffness larger and thus the time step required for stable solutions smaller. Equation (7) gives only an approximate value for the critical time increment. Upon encountering a numerically unstable situation, the time increment must be decreased further, until stable conditions are achieved. Looking at the time histories of vertical displacement computed using different time increments (Figure 11) it is apparent that seemingly stable behaviour does not mean really stable. According to equation (7) the time increment pertaining to the chosen value of the spring constants and the weight of the lightest block (1·4 kg) used would have been around 5×10^{-4} s. In order to select an appropriate time increment the time history of vertical displacement computed using $\Delta t = 1{\cdot}375 \times 10^{-7}$ was chosen as pivot. The sums of the squared differences in the vertical co-ordinates relative to the pivot time history were computed then, according to

$$\mathrm{d}y = \sum_{t=0\,\mathrm{s}}^{t=2\,\mathrm{s}} (y_{\Delta t_{\text{pivot}}}(t)) - y_{\Delta t_i}(t))^2 \tag{8}$$

where $y_{\Delta t_{\text{pivot}}}(t)$ is the pivot time history of vertical displacements of the oscillating block and $y_{\Delta t_i}(t)$ are the time histories for increasing values of the time increment, using nine different values of the time increment. Figure 12 shows the five smallest time increments versus the pertaining sums of squared co-ordinate differences. The stability of the computation seems to be lost beyond $3{\cdot}3 \times 10^{-6}$ s. On the basis of this curve $2{\cdot}0 \times 10^{-6}$ s seemed appropriate for stable computation.

Material parameters. Stiffness, damping and friction parameters for the DE model shown in Figure 7 were estimated as follows. Firstly, a wave propagation experiment was carried out to measure wave propagation velocities in the material of the specimens. As is shown in Figure 13, a small wooden parallelepiped was glued to one of the end-plates (right side in the photograph) of the specimen used in the wave propagation experiment. Four accelerometers were attached to the surface of the specimen as shown. To measure the propagation velocity of P and S waves, stress waves were generated in the specimen by means of a hammer. The attached parallelepiped was hit perpendicular to the end-plate of the specimen to measure P-wave propagation velocity and parallel with the end-plate to measure S-wave propagation velocity. From the records of acceleration the time instants of a particular crest of the propagating wave at all of the four accelerometers were noted and the time differences were computed. From the time differences and the distance between the accelerometers the wave propagation velocities were then evaluated. Having thus computed the P and S wave velocities it was already possible to derive[18] the corresponding normal and shear spring constants according to

$$k = m\frac{V^2}{\Delta X^2} \tag{9}$$

where V is the velocity of wave propagation of P and S waves in the wooden specimens. ΔX is the distance between the centres of two contacting elements and m is the mass of the element. Computed values of the propagation velocities for the exerted P and S waves were very close to one another and it was not possible to

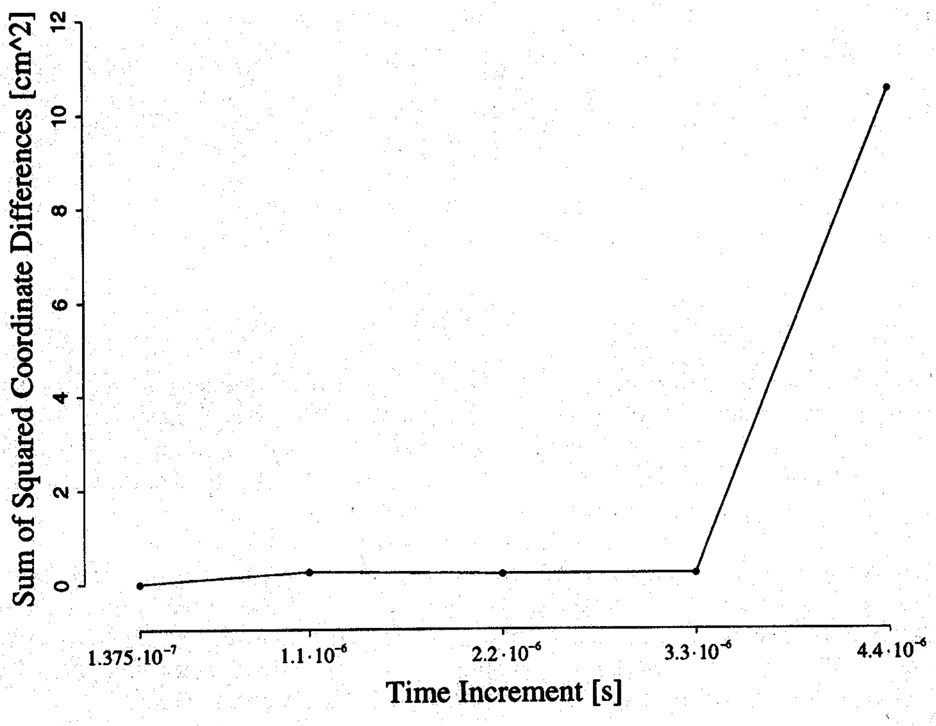


Figure 12. Differences in the vertical co-ordinates for the smallest five time increment values

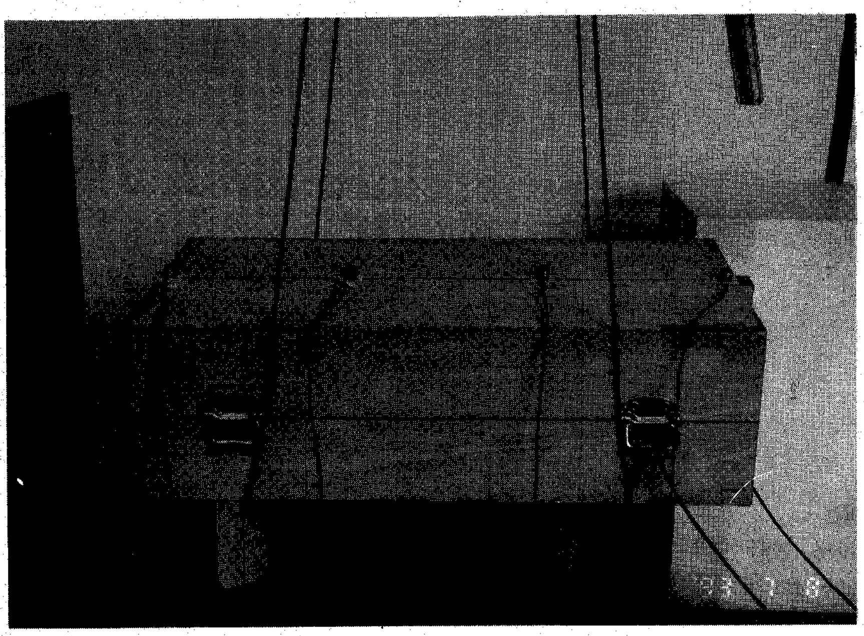


Figure 13. Wave propagation experiment on Block 1

distinguish between the discrepancies due to errors of the measurement and real differences. Therefore it was decided that the same value of spring constant ($k_n = k_s = 1{\cdot}98 \times 10^7$ N/m) would be used for both of the normal and shear springs throughout the course of the numerical simulation.

The energy dissipation at the contacts was represented by velocity proportional damping. Both of the normal and the shear damping parameters were assigned one and the same value. Shear damping was 'switched off' from the moment a block started sliding with regard to the energy dissipation due to friction. To identify an appropriate damping constant, a free-vibration test of a single block was performed. One of the base edges of the block was somewhat uplifted and then the block was released and started free vibrations. The base for comparing free vibration of a single block from the shaking table test, from the analytical study and from the DE simulation were time histories of horizontal displacement of the middle of the top of the specimen. Since the characteristics of free vibration, assuming that sliding does not occur, are unambiguously determined by the stiffness and damping coefficient, several computations were performed using the previously computed stiffness values and varied values of the damping coefficient. The damping coefficient which gave the closest coincidence of observed and numerically simulated time histories of horizontal displacement was deemed appropriate. As to how sensitive the results are to the values of the spring constant and the damping parameter it can be said that a variation of the spring constant influences mainly the frequency of rocking and the damping parameter mainly the amplitude. It was found that within a certain range of variation of the spring constant there is always a damping value to be found that will lead to a fairly close fitting between measured and numerically simulated time histories. Inferring from this it seems reasonable to say that the accurate establishment of the damping parameter is of greater importance than that of the spring constant. Best-fit time histories from the analytical computation and from the DE simulation together with the experimentally observed one are shown in Figure 5.

It is to be noted that in order to be able to compare directly the displacement time histories of free vibration the procedure for computing displacements from the two channels' records of acceleration was performed here. It was found that the convergence of the computation is dependent chiefly upon the exact knowledge of the initial angle of rotation of the pickups. The accuracy of the calibration of each pickup (voltage vs. acceleration) also is of primary concern. Since it was impossible to measure the initial angles to an accuracy that would have sufficed, those had to be altered many times before it became possible to achieve convergence in the case of this 7 s time history.

It would have been infeasible to perform this long iteration in the case of those 90 s channel records from the shaking table experiment since the initial angles of rotation were only theoretically and by no means practically equal to zero.

RESULTS

Criteria of rocking for single blocks

There are no significant differences in corresponding acceleration amplitude values between the results from Method 1 and Method 2. All of the criteria for rocking of single blocks are from Method 1.

The effect of size. Figure 14 contains four sets of criteria, one for each block. Qualitatively, all curves suggest that there is no significant difference in the amplitude of input base excitation to initiate rocking of rigid blocks of varied size and that initiation of rocking is dependent exclusively upon the breadth/height (b/h) ratio of the blocks. This is consistent with the results of other researchers; that is, when the input acceleration exceeds $b/h \cdot g$ (West's formula), rocking motion of the rigid block begins. Quantitatively, the results from the analytic study and from the DE simulation agree well. At many points of the criteria from the shaking table tests there is 10–20 per cent difference in acceleration amplitude with respect to the analytical or DE values. This can be explained by the extreme sensitivity of the system to the conditions between its base and the ground.[1,3,6,11,12] It was mentioned earlier that abrasive paper was attached to the lower base surfaces of all of the blocks in the shaking table tests to ensure sufficiently large friction between the blocks and the base. This could cause the edges to be not perfectly rectangular and thus contribute to such discrepancies. Another reason for the differences in the criteria is that even if the table's prescribed acceleration is purely horizontal, it undergoes slightly different 'pitching' motions and measurements of the response of a block exposed to the same table acceleration may significantly differ from each other (Yim *et al.*[6]).

The effect of slenderness. Criteria of the initiation of rocking for blocks of varied slenderness are shown in Figure 15. Again, as the input base acceleration attained the $b/h \cdot g$ value corresponding to the breadth/height ratio of a block, rocking initiated. The analytical criteria are almost perfectly constant while some of the criteria from the shaking tests show an inclination toward lower amplitude values as the frequency of excitation increases. It can be observed that slender blocks (Blocks 3 and 7) are more sensitive to their base conditions and to vertical disturbances of the shaking table than stocky ones (Blocks 5 and 6).

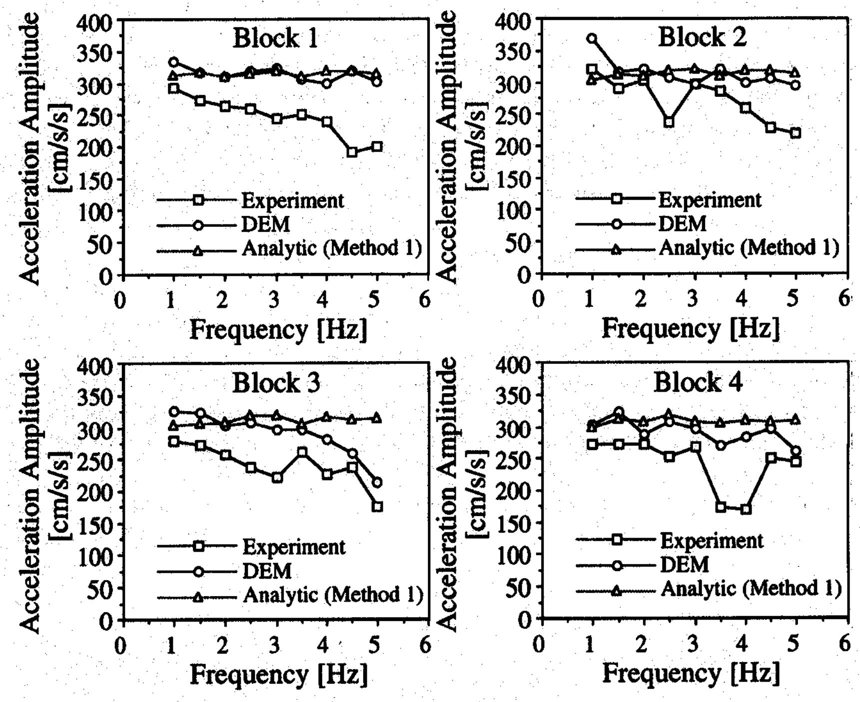


Figure 14. Criteria of rocking for Blocks 1, 2, 3, and 4

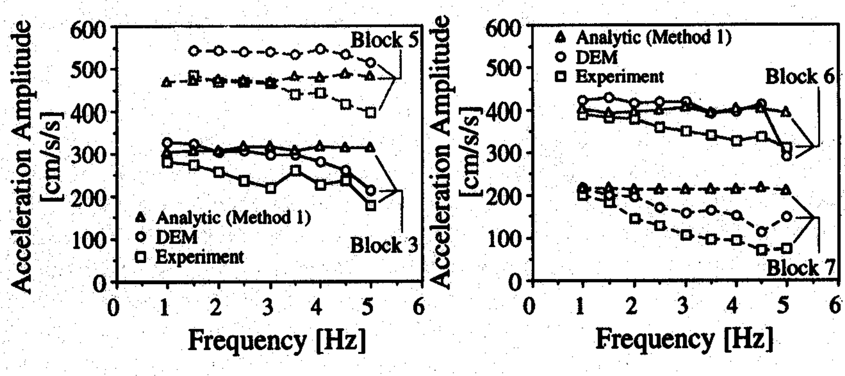


Figure 15. Criteria of rocking for Blocks 5, 6, 3 and 7

Analytically established (from Method 1) criteria for rocking of single blocks in Figures 14 and 15 are very nearly a straight line and the ordinates are close to the corresponding theoretical $b/h \cdot g$ values. An additional reason for not being entirely straight is inherent in the nature of the input sinusoidal base motions that were used. These input motions were recorded in the shaking table experiments. It was attempted to make the envelope of the linearly increasing amplitude of the sinusoidal base motion a straight line but it was impossible to maintain a constant slope of the envelope. These small changes in the fashion of the 'harmonic' excitation may well evoke a somewhat earlier or delayed initiation of the rocking motion.

As for the practical use of the curves in Figures 14 and 15 it can be said that these criteria were the first and the most elementary point of reference to the correctness of the DE computation the authors could find. The rocking of single blocks has been extensively studied and covered in the technical literature. Criteria for the initiation of rocking are available unlike those for overturning. The purpose of comparison was not to discover some yet unknown nature of the phenomenon of rocking but to have a well-established point the DEM could be tested at. The comparison of the curves provided useful suggestions to the establishment of contact parameters and future shaking table tests.

Criteria of overturning for single blocks

Criteria of overturning for single blocks (Blocks 1, 3 and 7) from the two analytical methods and from the DE simulation are shown in Figure 16. All of the three sets of curves show that the characteristics of the criteria from Method 2 and from the DE simulation are very similar. The criteria from Method 1 are highly

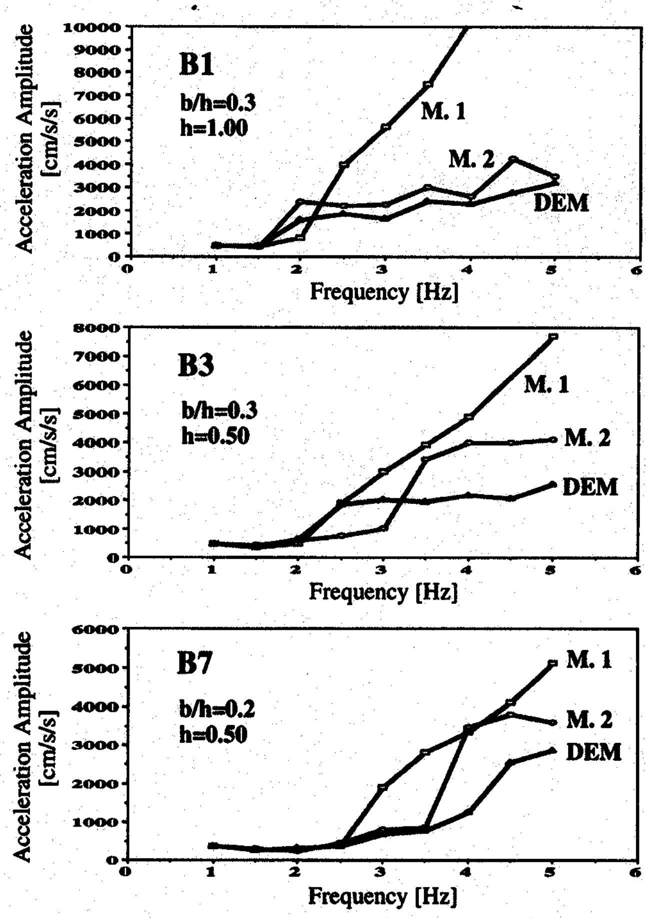


Figure 16. Criteria of overturning for single blocks (Blocks 1, 3 and 7)

inconsistent with the other two, showing an almost linear increase of overturning amplitude with an increase in the frequency of excitation. Method 1 does not allow for the consideration of non-linear effects such as sliding or eventual lift-off. The body is supposed to respond corresponding to only one motion pattern which is rotation about the base edges. It is clearly manifested that Method 2 was a much better choice than Method 1 although there still are, at a few frequencies even large, quantitative differences as compared with results from the DE simulation. It was found that the motion pattern according to which the block moves at large input amplitudes is mostly sliding rotation and, for short intervals of time, rotational jump.

Owing to the limitations of capacity of the shaking table used in the experiment there were only a limited number of examples for overturning of single blocks. These were at low frequency excitations where one cannot talk about both rocking and overturning since rocking does not form but the specimens overturn at the moment rocking would initiate. At higher frequencies, one cannot name values of frequency unless the b/h ratio of the block in question is added, where overturning is preceded by a longer period of rocking the acceleration that would be required for overturning is generally much larger than those allowed for by the shaking table that was used.

Block assemblies

Results for the tested two-block columns from both the shaking table test and the DE simulation confirm that, at frequencies under 2·0 Hz, the freely standing column, denoted by F1 and F2 in Figure 17, loses stability before the one set next to a wall, denoted by W1 and W2. Above 2·0 Hz the column that interacts

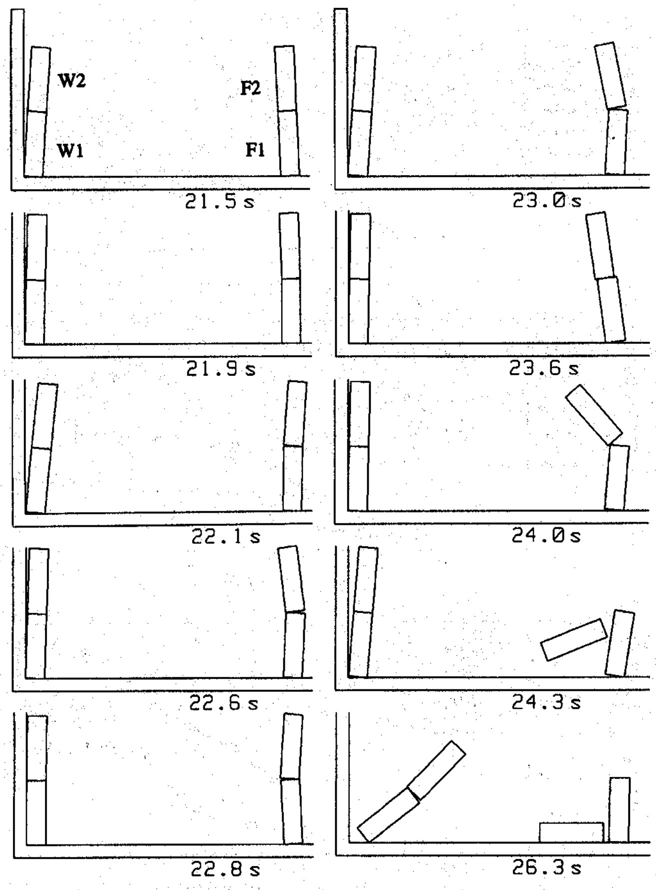


Figure 17. Characteristic frames of the animation of the response of a freely standing two-block column under a 3 Hz sinusoidal base motion

with a wall loses stability first and turns over. This means that in order for one to tell whether it is recommended to place furniture right next to a wall or, if not, the width of the gap in between, one ought at least to know approximately the frequency of the ground motion the system will be exposed to. As for the agreement between results from the shaking table test and from the DE analysis (Figures 18 and 19), it can be said that, with the exception of a few cases that are in the frequency range above 4·0 Hz, it is reasonably good both qualitatively and quantitatively. The overall behaviour of the assemblies, judged by comparing video records and real time animation of response (Figure 17), are very similar.

An examination of Figures 17 and 20 may allow for a somewhat deeper insight into the nature of the response of the tested two-block assemblies. Figure 20 shows a segment of the time histories of the angular displacements of the freely standing two-block column under the shown 3 Hz sinusoidal base excitation. Figure 17 gives a number of squares of the animation of the response. From both of the figures it is clear that the two blocks rocked together before an amplitude of input acceleration was attained (around 21 s) and then after separation they rocked under different angles until they lost stability. At lower frequencies there is hardly any separation; indeed, the assembly loses stability as one block. With an increase in the frequency of the excitation the separation occurs earlier and earlier and there is a period of rocking as two separate blocks before losing stability. These figures together with the pertaining criteria for overturning show how well the DE model was able to follow the very highly non-linear interaction of two rigid blocks.

The column of three blocks was the most complicated assembly tested in the shaking table experiment. Figure 21 shows the loss of stability of the column in the shaking table tests in response to harmonic base motion. Representative moments of the animation under a 3 Hz harmonic base motion are shown in Figure 22. The criteria of overturning under harmonic excitation from the experiment and from the DE simulation are shown in Figure 23. As with the cases of two-block columns, reasonably good agreement was found. The DE

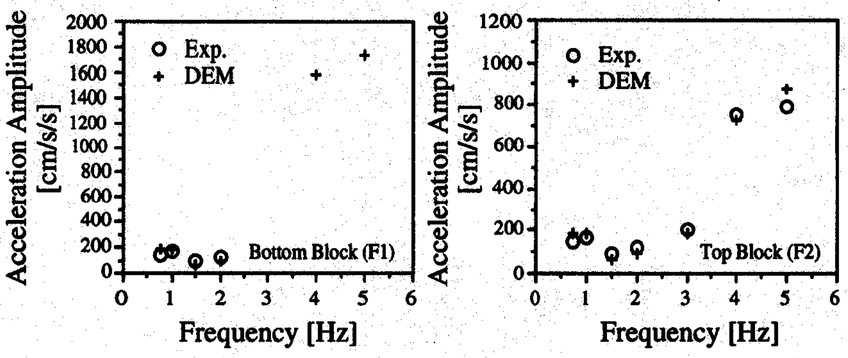


Figure 18. Criteria of overturning for a freely standing column of two blocks

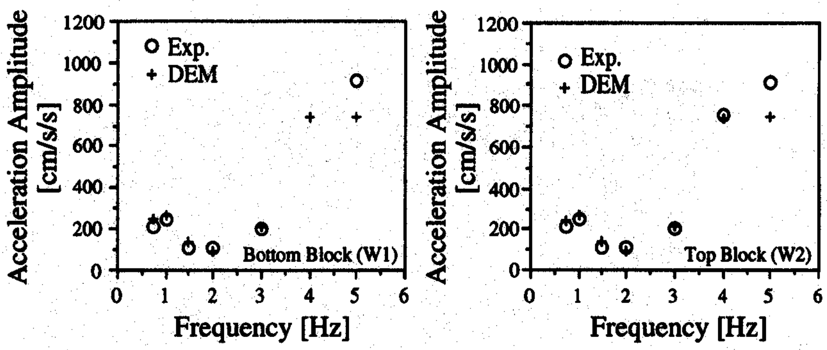


Figure 19. Criteria of rocking for a column of two blocks interacting with a wall

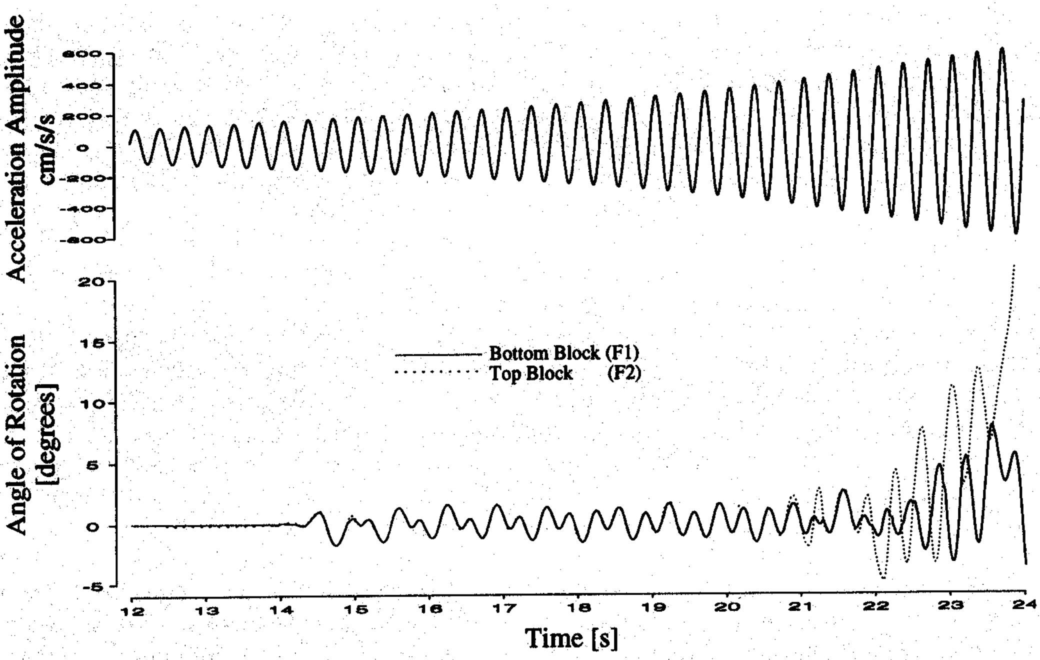


Figure 20. A segment of the 3 Hz horizontal input motion and the response angular displacements of a freely standing two-block column

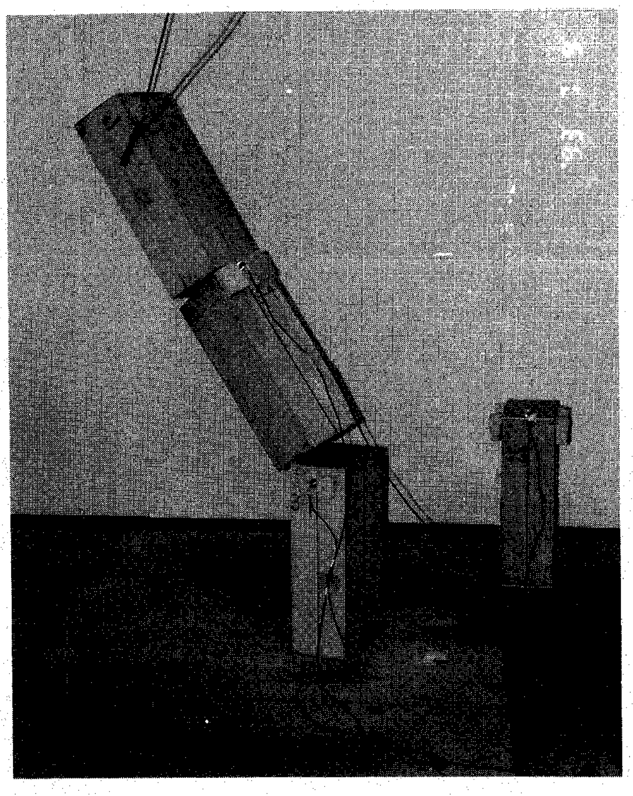


Figure 21. Loss of stability of a column of three blocks under a 3 Hz harmonic base motion

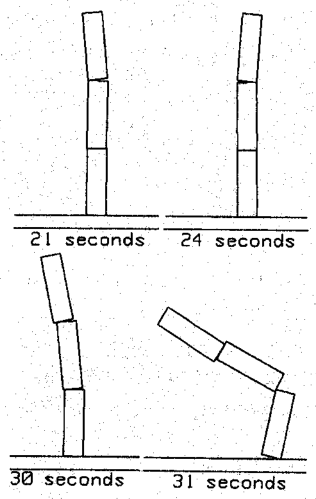


Figure 22. Instants of the animation of the response of a column of three blocks under a 3 Hz harmonic base motion

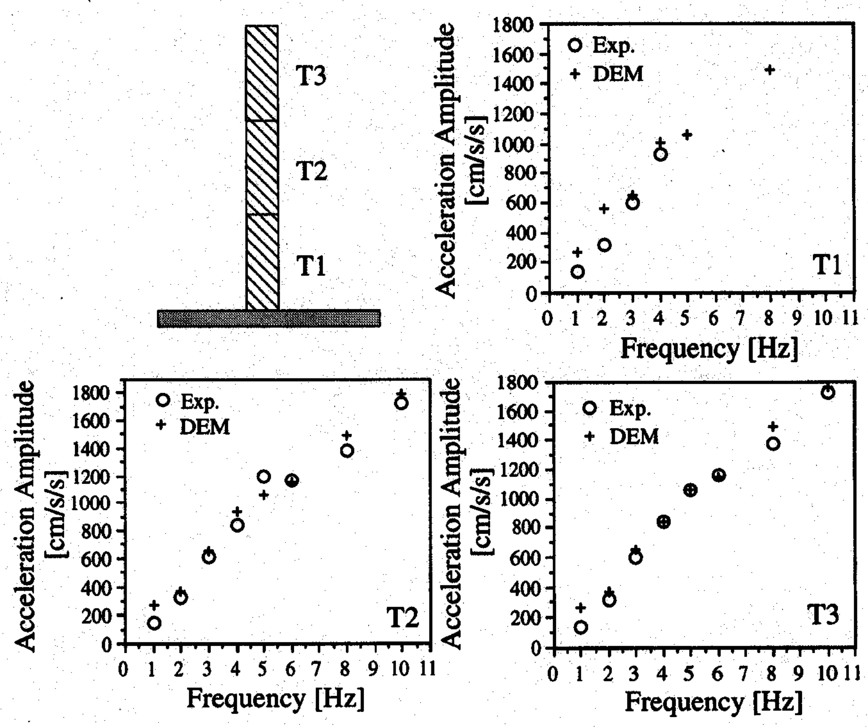


Figure 23. Criteria of overturning for a column of three blocks

model in use seemed to be able to follow the interactive, highly non-linear response of this complex assembly quite accurately.

Discrepancies between overturning acceleration amplitudes may be attributed to the phenomenon in which the blocks, in response to a high-frequency excitation, turn around their vertical axes, that is, the response is no longer limited to take place in the plane of the applied base motion, but also in a direction perpendicular to it. Thus, the base acceleration value detected at the moment a block overturns can be misleading as the conditions of the shaking table tests can no longer be considered two-dimensional.

It is rather regrettable that, due to the nature of technical papers, it is not possible to present the animation in contrast with the video recordings from the experiment which would be inevitably much more convincing.

CONCLUSION

Shaking table tests on rigid, rectangular blocks and block assemblies are performed. Then analytical studies of criteria of rocking and of overturning for single blocks are conducted. Finally, distinct element simulation of the response of single blocks and of block assemblies is performed. Satisfactory agreement is found between the criteria of rocking from the analytical study and from the DE analysis. Quantitative differences between criteria of rocking from the shaking table tests and the corresponding numerical and analytical results are within a reasonable range. Criteria of overturning for single blocks from Method 2 and from the DE simulation show good qualitative agreement. There are large differences between input acceleration amplitudes at frequencies beyond 3 Hz. Regarding overturning of the tested block assemblies, quantitative differences in the results from the shaking tests and from the DE simulation are rather small. Qualitatively, the distinct element model can be said to have performed well. Quantitative agreement with the test results is good, with the exception of a limited number of points, which are generally in the range of higher frequencies (above 4 Hz).

Continuing the present study the authors intend to perform the numerical simulation of those shaking table tests in which natural earthquake excitations were applied. Also, a video-assisted system to measure displacements of corners of the specimens will be installed to facilitate a more accurate comparison of the results.

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