Digital Generation of Non-Gaussian Stochastic Fields

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Abstract: A method by which sample fields of a multidimensional non-Gaussian homogeneous stochastic field can be generated is developed. The method first generates Gaussian sample fields and then maps them into non-Gaussian sample fields with the aid of an iterative procedure. Numerical examples indicate that the procedure is very efficient and generated sample fields satisfy the target spectral density and probability distribution function accurately. The proposed method has a wide range of applicability to engineering problems involving stochastic fields where the Gaussian assumption is not appropriate.

Introduction

The methods of digitally generating sample functions of stationary and nonstationary stochastic processes have been well established in the last two decades or so on the basis of the spectral or autoregressive moving-average (ARMA) representation of stochastic processes. The same observation can be made for the methods of generating sample functions of homogeneous or nonhomogeneous stochastic fields.

These models have been extensively used in various applications in the areas of engineering mechanics and structural engineering. For example, the methods are frequently used for the purpose of Monte Carlo solution of a class of structural response problems; typical examples are the random vibration analysis of nonlinear structures subjected to earthquake acceleration, ocean waves, and boundary layer turbulence, and of building structures parametrically excited by wind forces, where all these excitations are idealized as stochastic processes.

More recently, the problems of system stochasticity have attracted considerable attention and interest among engineers and researchers. In this respect, the theory of stochastic fields has been advanced (e.g., Shinozuka 1985; Vanmarcke 1983) and the methods of digitally generating sample functions again for the purpose of Monte Carlo solution of finite element analysis problems involving stochastic fields have been developed (e.g., Yamazaki et al. 1986).

However, because of mathematical difficulties and the lack of observed data, most generation techniques as well as analytical approaches are limited to dealing with Gaussian fields. In some situations, such a Gaussian assumption is not appropriate because the observed fields exhibit non-

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Gaussian characteristics. For example, even if the input motions are Gaussian, the output responses of nonlinear systems are known to be non-Gaussian. In the field of ocean engineering, Ochi (1986) showed that the wave height records in a sea of finite water depth have a skewed non-Gaussian nature. Also, the spatial variability of Young's modulus and some other material properties should not be considered to form Gaussian fields since theoretically they are not supposed to take negative values. Therefore, the development of a method for generating sample functions of non-Gaussian fields appears to have theoretical as well as practical significance.

As one of the few studies in this direction involving non-Gaussian processes, Shinozuka and Tan (1981) demonstrated such a generation method in which the prescribed crossing rate is reproduced by mapping a Gaussian process into a non-Gaussian process while reproducing the prescribed irregularity ratio in approximation at the same time. The same procedure was applied to the generation of load histories by Wilkins et al. (1975) for the purpose of testing the fatigue performance of aircraft structures. Grigoriu (1984) also provided an approximate analytical method for the evaluation of crossing rates on non-Gaussian processes.

In the present paper, a more general generation method is developed for non-Gaussian multidimensional homogeneous fields. The method utilizes an iterative procedure involving the generation method for Gaussian homogeneous fields on the basis of their spectral representation, and the mapping technique which transforms Gaussian sample functions into non-Gaussian sample functions on the probability distribution function diagram.

Simulation of Gaussian Stochastic Fields

There are several ways in which Gaussian homogeneous stochastic fields are simulated and their sample functions generated; for example, (1) In terms of trigonometric series on the basis of spectral representation of fields; (2) by means of covariance decomposition; and (3) with the aid of ARMA models. The first method which has been extensively studied by Shinozuka and his associates (Shinozuka and Jan 1972; Shinozuka 1974, 1985) is described in detail herein in order to facilitate the introduction of simulation and generation methods for non-Gaussian fields in the sections that follow. Although the methods are valid for general multidimensional fields, two-dimensional fields are considered here for clarity of presentation.

First, the spectral density function $S_{ff}(\cdot)$ or autocorrelation function $R_{ff}(\cdot)$ of the two-dimensional stochastic field is assumed to be given. These functions consist of a pair of Wiener-Khintchine relationships as

$$S_{ff}(\kappa_x, \kappa_y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{ff}(\xi_x, \xi_y) e^{-i(\kappa_x \xi_x + \kappa_y \xi_y)} \, d\xi_x \, d\xi_y \quad \ldots \ldots \ldots (1)$$

$$R_{ff}(\xi_x, \xi_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{ff}(\kappa_x, \kappa_y) e^{i(\kappa_x \xi_x + \kappa_y \xi_y)} \, d\kappa_x \, d\kappa_y \quad \ldots \ldots \ldots (2)$$

in which κ_x and κ_y = wave numbers; and ξ_x and ξ_y = separation distances in two directions. The zero-mean stochastic field $f(x, y)$ can be simulated (Shinozuka 1985) by

$$f(x, y) = \sqrt{2} \sum_{k=1}^{N_x} \sum_{l=1}^{N_y} A_{kl} [\cos(\kappa_{xk} x + \kappa_{yl} y + \phi_{kl}) + \cos(\kappa_{xk} x - \kappa_{yl} y + \psi_{kl})] \quad \ldots \ldots \ldots (3)$$

with

$$A_{kl} = \sqrt{2 S_{ff}(\kappa_{xk}, \kappa_{yl}) \Delta\kappa_x \Delta\kappa_y} \quad \ldots \ldots \ldots (4)$$

in which quadrant symmetry of the spectral density with respect to the origin is assumed. Also, ϕ_{kl} and ψ_{kl} are independent random phase angles uniformly distributed between 0 and 2π and

$$\Delta\kappa_x = \frac{\kappa_{xu}}{N_x} \quad \ldots \ldots \ldots (5a)$$

$$\Delta\kappa_y = \frac{\kappa_{yu}}{N_y} \quad \ldots \ldots \ldots (5b)$$

$$\kappa_{xk} = k \Delta\kappa_x \quad \ldots \ldots \ldots (5c)$$

$$\kappa_{yl} = l \Delta\kappa_y \quad \ldots \ldots \ldots (5d)$$

where κ_{xu} and κ_{yu} = the upper cut-off wave numbers in the first quadrant of the $\kappa_x - \kappa_y$ plane.

The digital generation of sample functions of Eq. 3 is readily computed with the aid of the fast Fourier transform (FFT) algorithm by rewriting Eq. 3 in the following form:

$$f(x_p, y_q) = Re\left[\sqrt{2} \sum_{k=0}^{M_x-1} \left(\sum_{l=0}^{M_y-1} A_{kl} e^{i\phi_{kl}} e^{i\kappa_{yl} y_q} \right) e^{i\kappa_{xk} x_p}\right] + Re\left[\sqrt{2} \sum_{k=0}^{M_x-1} \left(\sum_{l=0}^{M_y-1} A_{kl} e^{i\psi_{kl}} e^{-i\kappa_{yl} y_q} \right) e^{i\kappa_{xk} x_p}\right] \quad \ldots \ldots \ldots (6)$$

in which $Re[\cdot]$ represents the real part and

$$x_p = p \Delta\xi_x \qquad (p = 0, 1, \ldots, M_x - 1) \quad \ldots \ldots \ldots (7a)$$

$$y_q = q \Delta\xi_y \qquad (q = 0, 1, \ldots, M_y - 1) \quad \ldots \ldots \ldots (7b)$$

Here, N_x, N_y, M_x, and M_y must all be positive integer powers of two and $M_x > N_x$ and $M_y > N_y$. Also,

$$\Delta\xi_x = \frac{2\pi}{\Delta\kappa_x M_x} = \frac{2\pi N_x}{\kappa_{xu} M_x} \quad \ldots \ldots \ldots (8a)$$

$$\Delta\xi_y = \frac{2\pi}{\Delta\kappa_y M_y} = \frac{2\pi N_y}{\kappa_{yu} M_y} \quad \ldots \ldots \ldots (8b)$$

It is noted that $f(x_p, y_q)$ is periodic with periods L_x and L_y in two directions as

$$L_x = M_x \Delta\xi_x = \frac{2\pi}{\Delta\kappa_x} \quad (9a)$$

$$L_y = M_y \Delta\xi_y = \frac{2\pi}{\Delta\kappa_y} \quad (9b)$$

The identity between Eqs. 3 and 6 is obtained if A_{kl} in Eq. 6 is set as

$$A_{kl} = 0 \quad (\text{if} \quad k = 0 \quad \text{or} \quad l = 0) \quad (10)$$

and

$$A_{kl} = 0 \quad (\text{if} \quad k \geqq N_x + 1 \quad \text{or} \quad l \geqq N_y + 1) \quad (11)$$

Eq. 10 is necessary for the stochastic field to have mean-zero with respect to x or y or both directions. For Eq. 11 to be valid, the cut-off wave numbers must be chosen in such a way that the spectral density is insignificant outside the domain indicated in the same equation. The simulated process is Gaussian by virtue of the central limit theorem.

Evaluation of Spectral Density from Sample Fields

The method of evaluating the spectral density from a sample field is described for later purposes. Assuming ergodicity, the autocorrelation function is defined by

$$R_{ff}(\xi_x, \xi_y) = E[f(x, y)f(x + \xi_x, y + \xi_y)]$$

$$= \lim_{L_x, L_y \to \infty} \frac{1}{L_x L_y} \int_0^{L_x} \int_0^{L_y} f(x, y) f(x + \xi_x, y + \xi_y)\, dx\, dy \quad (12)$$

The Fourier transform of $f(x, y)$ is defined by

$$F(\kappa_x, \kappa_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-i(\kappa_x x + \kappa_y y)}\, dx\, dy \quad (13)$$

Also, the inverse Fourier transform is represented by

$$f(x, y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(\kappa_x, \kappa_y) e^{i(\kappa_x x + \kappa_y y)}\, d\kappa_x\, d\kappa_y \quad (14)$$

For the sample function $f(x, y)$ defined in the range

$$0 \leqq x \leqq L_x \quad \text{and} \quad 0 \leqq y \leqq L_y \quad (15)$$

Eq. 13 can be rewritten as

$$F(\kappa_x, \kappa_y) = \int_0^{L_x} \int_0^{L_y} f(x, y) e^{-i(\kappa_x x + \kappa_y y)}\, dx\, dy \quad (16)$$

Introducing Eq. 12 into Eq. 1 and utilizing Eq. 16,

$$S_{ff}(\kappa_x, \kappa_y) = \lim_{L_x, L_y \to \infty} \frac{1}{(2\pi L_x)} \frac{1}{(2\pi L_y)} F(\kappa_x, \kappa_y) F(-\kappa_x, -\kappa_y) \quad (17)$$

If $f(x, y)$ is a real process, the following relationship is obtained from Eq. 13 as

$$F(-\kappa_x, -\kappa_y) = F^*(\kappa_x, \kappa_y) \quad (18)$$

in which the asterisk denotes the complex conjugate. Then, Eq. 17 reduces to

$$S_{ff}(\kappa_x, \kappa_y) = \lim_{L_x, L_y \to \infty} \frac{1}{(2\pi L_x)} \frac{1}{(2\pi L_y)} |F(\kappa_x, \kappa_y)|^2 \quad (19)$$

The discretized form of Eq. 16 can be derived by utilizing Eqs. 7–9 as

$$F(\kappa_{xk}, \kappa_{yl}) = \Delta\xi_x \Delta\xi_y \sum_{p=0}^{M_x - 1} \sum_{q=0}^{M_y - 1} f(x_p, y_q) e^{-(\kappa_{xk} x_p + \kappa_{yl} y_q)}$$

$$(k = 0, 1, \ldots, M_x - 1; \quad l = 0, 1, \ldots, M_y - 1) \quad (20)$$

Utilizing Eqs. 9 and 20, the discretized version of Eq. 19 is derived as

$$S_{ff}(\kappa_{xk}, \kappa_{yl}) = \frac{1}{(2\pi L_x)} \frac{1}{(2\pi L_y)} |F(\kappa_{xk}, \kappa_{yl})|^2$$

$$= \frac{1}{\Delta\kappa_x \Delta\kappa_y} \left| \frac{1}{M_x M_y} \sum_{p=0}^{M_x - 1} \sum_{q=0}^{M_y - 1} f(x_p, y_q) e^{-i(\kappa_{xk} x_p + \kappa_{yl} y_q)} \right|^2 \quad (21)$$

Eq. 21 can readily be calculated by using the FFT algorithm.

Digital Generation of Non-Gaussian Stochastic Fields

A method for digitally generating sample functions of multidimensional non-Gaussian homogeneous stochastic fields is developed. The flow chart of the method is shown in Fig. 1.

First, the target spectral density $S_{bb}^T(\kappa)$ and the one-dimensional probability distribution function $F_B(b)$ of the non-Gaussian stochastic field are specified. Here, $\boldsymbol{\kappa} = [\kappa_x\ \kappa_y \ldots]^T$ is a wave number vector of an appropriate dimension, and $F_B(b)$ denotes a one-dimensional distribution function of a random variable b representing a non-Gaussian stochastic field with zero mean and variance σ_b^2. Correspondingly, one can easily find the Gaussian probability distribution function $F_G(g)$ with zero mean and variance $\sigma_g^2 = \sigma_b^2$.

The first step of the method is to assume the spectral density $S_{gg}^{(1)}(\boldsymbol{\kappa})$ of the Gaussian field to be

$$S_{gg}^{(1)}(\boldsymbol{\kappa}) = S_{bb}^{(T)}(\boldsymbol{\kappa}) \quad (22)$$

and generate a sample function of the Gaussian field $g^{(1)}(\mathbf{X})$ by Eq. 6. Here, $\mathbf{X} = [x, y, \ldots]^T$ denotes a position vector in the space with an appropriate

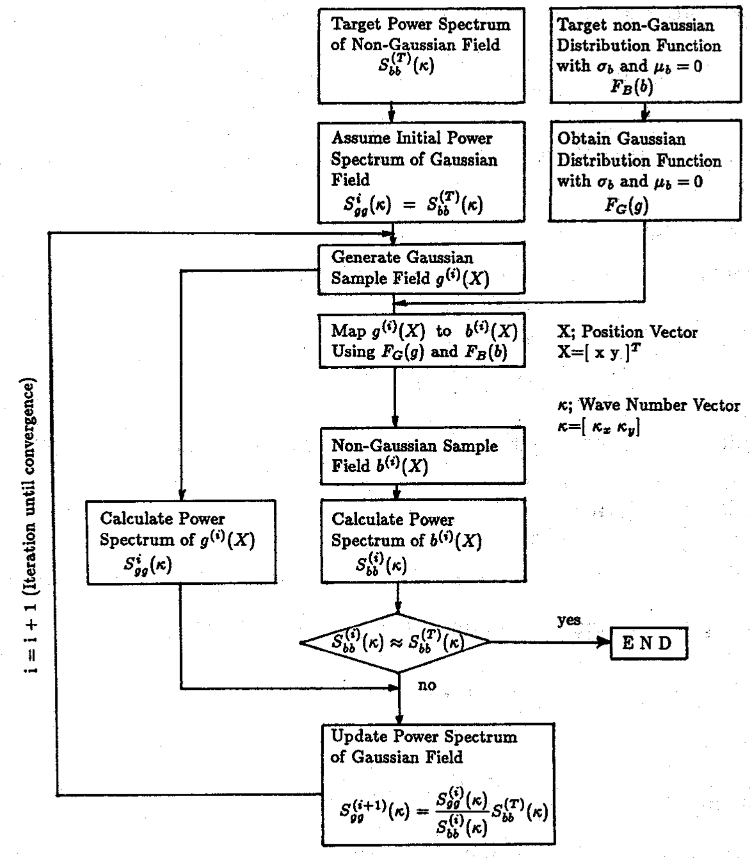

FIG. 1. Flow Chart of Digital Generation of Non-Gaussian Stochastic Fields

dimension. The simulated field $g^{(1)}(\mathbf{X})$ is transformed into the non-Gaussian field $b^{(1)}(\mathbf{X})$ by a mapping technique; the generated value $g^{(1)}(\mathbf{X}_k)$ at $\mathbf{X}_k$ ($k = 1, 2, \ldots, M_xM_y$) is mapped into

$$b^{(1)}(X_k) = F_B^{-1}\{F_G[\,g^{(1)}(X_k)]\} \quad (23)$$

on the probability distribution function diagram. If the number of data points is large enough, Eq. 23 generates the sample field with the target non-Gaussian distribution function.

However, the spectral density of $S_{bb}^{(1)}(\mathbf{X})$ of $b^{(1)}(\mathbf{X})$ calculated by Eq. 21 does not coincide with the target spectral density $S_{bb}^{(T)}(\mathbf{X})$, because the transformation represented by Eq. 23 is nonlinear. Therefore, the iterative algorithm is introduced in order to satisfy the target spectral density. The spectral density for generating the Gaussian stochastic field in the second iteration is assumed as

$$S_{gg}^{(2)}(\boldsymbol{\kappa}) = \frac{S_{gg}^{(1)}(\boldsymbol{\kappa})}{S_{bb}^{(1)}(\boldsymbol{\kappa})} S_{bb}^{(T)}(\boldsymbol{\kappa}) \quad (24)$$

or generally, the spectral density for generating sample functions of the Gaussian stochastic field in the $(i + 1)$th iteration is assumed to be

$$S_{gg}^{(i+1)}(\boldsymbol{\kappa}) = \frac{S_{gg}^{(i)}(\boldsymbol{\kappa})}{S_{bb}^{(i)}(\boldsymbol{\kappa})} S_{bb}^{(T)}(\boldsymbol{\kappa}) \quad (25)$$

Following the same procedure as used in the first iteration, the Gaussian field of the ith iteration $g^{(i)}(\mathbf{X})$ is generated using the updated $S_{gg}^{(i)}(\boldsymbol{\kappa})$ and the same random phase angles. The mapping into the non-Gaussian field is carried out by the generalized form of Eq. 23 as

$$b^{(i)}(\mathbf{X}_k) = F_B^{-1}\{F_G[\,g^{(i)}(\mathbf{X}_k)]\} \quad (26)$$

Updating the spectral density for the Gaussian field in terms of Eqs. 24 and 25 is performed for the following reasons. Through the nonlinear mapping represented by Eqs. 23 and 26, the spectral density for each wave number changes. However, the rate of change at each wave number which is represented by the ratio $S_{gg}^{(i)}(\boldsymbol{\kappa})/S_{bb}^{(i)}(\boldsymbol{\kappa})$ is expected to approach a constant value in a few iterations. In order to force $S_{bb}^{(i+1)}(\boldsymbol{\kappa})$ to approach $S_{bb}^{(T)}(\boldsymbol{\kappa})$ as i increases, Eq. 25 is devised. This iterative algorithm has been proven to work very efficiently in the following numerical example.

NUMERICAL EXAMPLE

A numerical example of the method described above involving a two-dimensional non-Gaussian homogeneous field is given. As a typical example of a non-Gaussian distribution, the beta distribution is assumed which has the following probability density function:

$$f_B(b) = \frac{1}{B(q, r)} \frac{(b - b_{\min})^{q-1}(b_{\max} - b)^{r-1}}{(b_{\max} - b_{\min})^{q+r-1}} \qquad (b_{\min} \leqq b \leqq b_{\max})$$

$$= 0 \qquad \text{(otherwise)} \quad (27)$$

in which $b_{\min}$ and $b_{\max}$ = the lower and upper bounds of the distribution; q and r = parameters of the distribution; and $B(q, r)$ is the beta function defined by

$$B(q, r) = \int_0^1 x^{q-1}(1 - x)^{r-1}\,dx = \frac{\Gamma(q)\Gamma(r)}{\Gamma(q + r)} \quad (28)$$

where $\Gamma(\,\cdot\,)$ is the gamma function. The probability distribution function corresponding to Eq. 27 is obtained as

$$F_B(b) = \frac{B_u(q, r)}{B(q, r)} \qquad (b_{\min} \leqq b \leqq b_{\max}) \quad (29)$$

in which $B_u(q, r)$ is the incomplete beta function defined by

$$B_u(q, r) = \int_0^u x^{q-1}(1-x)^{r-1}\,dx \quad \ldots\ldots \quad (30)$$

with u given by

$$u = \frac{(b - b_{\min})}{(b_{\max} - b_{\min})} \quad \ldots\ldots \quad (31)$$

The mean and variance of the beta distribution are known as

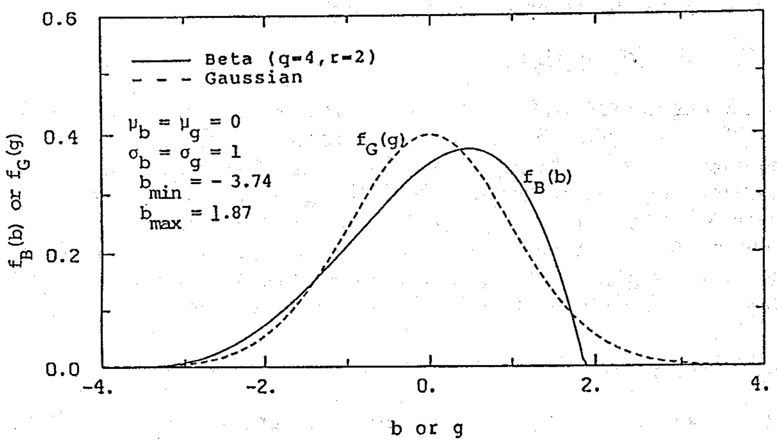

FIG. 2. Target Probability Density Functions

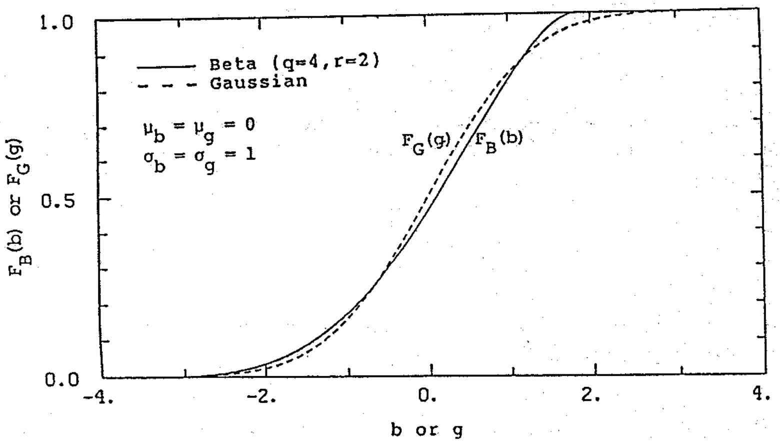

FIG. 3. Target Probability Distribution Functions

$$\mu_b = b_{\min} + \frac{q}{q+r}(b_{\max} - b_{\min}) \quad \ldots\ldots \quad (32)$$

$$\sigma_b^2 = \frac{qr(b_{\max} - b_{\min})^2}{(q+r)^2(q+r+1)} \quad \ldots\ldots \quad (33)$$

Hence, there are four parameters, q, r, $b_{\min}$ and $b_{\max}$, in Eq. 27. However, assuming μ_b and σ_b^2 and utilizing Eqs. 32 and 33, $b_{\min}$ and $b_{\max}$ can be written as

$$b_{\min} = \mu_b - \sigma_b\sqrt{\frac{q(q+r+1)}{r}}$$

$$b_{\max} = \mu_b - \sigma_b\sqrt{\frac{r(q+r+1)}{q}} \quad \ldots\ldots \quad (34)$$

The shape of the density function can be controlled by choosing appropriate values for q and r. $q = 4$ and $r = 2$ are adopted in the present study in order to consider a skewed distribution. Also, $\mu_b = 0$ and $\sigma_b = 1.0$ are assumed. The probability density function and the probability distribution function of the beta distribution are shown in Figs. 2 and 3 along with those of the Gaussian distribution of zero mean and unit standard deviation.

The spectral density function or autocorrelation function is also necessary to prescribe the stochastic field. The following isotropic spectral density function is assumed as the target without loss of generality.

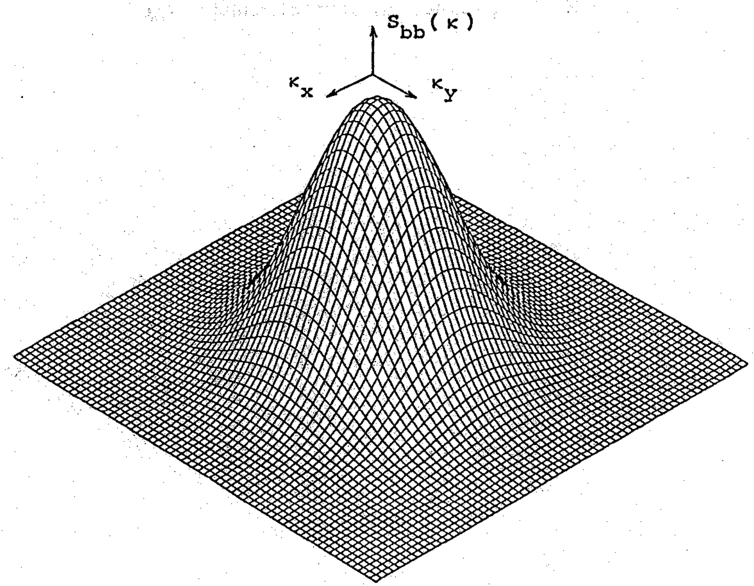

FIG. 4. Target Spectral Density Function

$$S_{bb}(\boldsymbol{\kappa}) = \sigma_b^2 \frac{d^2}{4\pi} e^{-d^2|\boldsymbol{\kappa}|^2/4} \qquad (35)$$

in which d = a parameter representing the decay of the correlation and $\boldsymbol{\kappa} = [\kappa_x \ \kappa_y]^T$ is a wave number vector. The corresponding autocorrelation function is

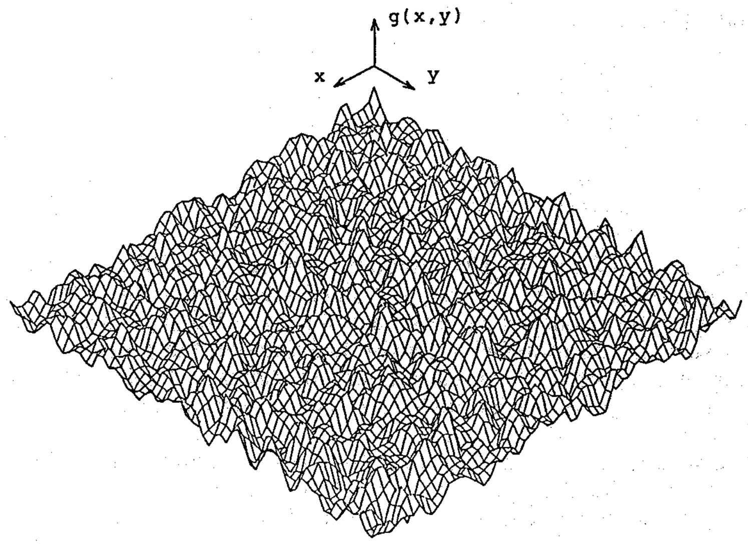

FIG. 5. Simulated Gaussian Stochastic Field

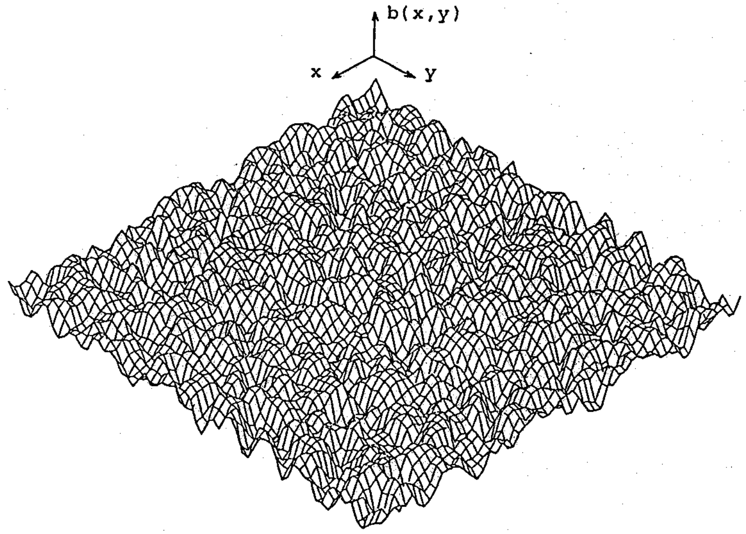

FIG. 6. Simulated Beta Stochastic Field

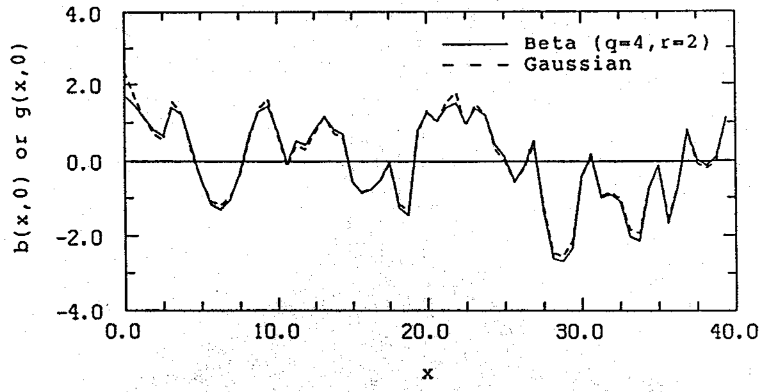

FIG. 7. Simulated Stochastic Fields Along $y = 0$

$$R_{bb}(X) = \sigma_b^2 e^{-|\xi|^2/d^2} \qquad (36)$$

in which $\boldsymbol{\xi} = [\xi_x \ \xi_y]^T$ = the separation vector.

The target spectral density is shown in Fig. 4 in a discretized form with constants $d = 1.0$, $\kappa_{xu} = \kappa_{yu} = 1.6\pi$, $N_x = N_y = 32$. The Gaussian stochastic field is generated by Eq. 6 with the number of data points equal to 4,096 ($M_x = M_y = 64$). By following the mapping and iterative procedure, the stochastic field of the beta distribution is obtained.

The Gaussian and beta stochastic fields thus generated at the end of three iterations are shown in Figs. 5 and 6. They look quite similar. However, the effect of nonlinear mapping can be observed if the fields along the line $y = 0$ are plotted as shown in Fig. 7.

The spectral density functions calculated from these fields are shown in Figs. 8 and 9. The spectral density of the beta field after three iterations is very close to the target. Since the original spectral density is transformed iteratively by Eq. 25, the initial smoothness of the spectral density of the Gaussian field is lost.

Because of the ergodicity of the generated fields, the statistical distributions of the 4,096 data points for the third iteration are plotted in Figs. 10 and 11 along with their target analytical forms. It is observed that the beta distribution is generated with high accuracy as well as the Gaussian distribution. Therefore, the simulated sample field is proved to satisfy both the assumed spectral density and probability distribution function.

The proposed generation method can be applied to any form of the distribution functions so long as it is given analytically or at least numerically. Also, a similar mapping and iterative procedure can be applied to other generation methods, such as covariance matrix decomposition methods (e.g., Shinozuka 1987). In this case, the sample generation is based on the covariance matrix resulting from the autocorrelation function. Therefore, the covariance matrix should be updated in a form similar to that of Eq. 25 by replacing the spectral densities with covariances. In conjunction with the statistical preconditioning algorithm (Yamazaki and Shinozuka 1986), such a generation of sample functions of

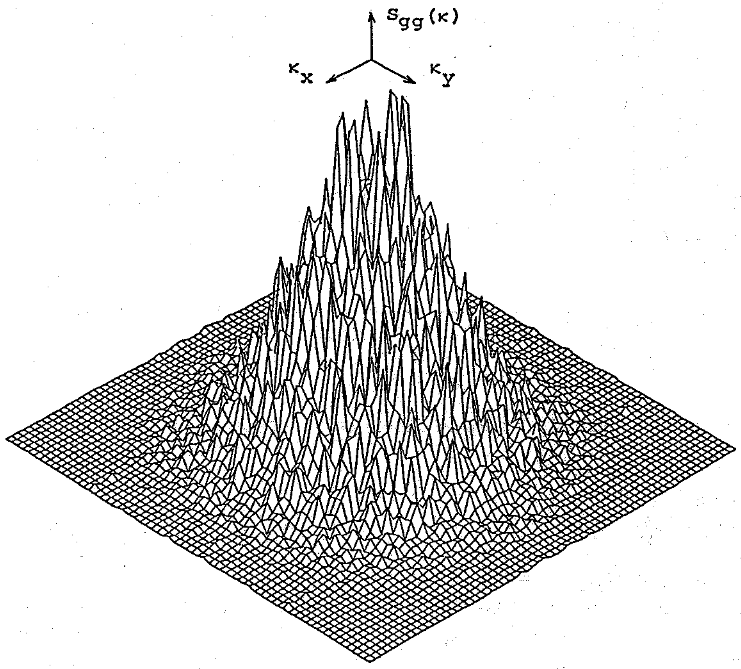

FIG. 8. Spectral Density of Simulated Gaussian Field

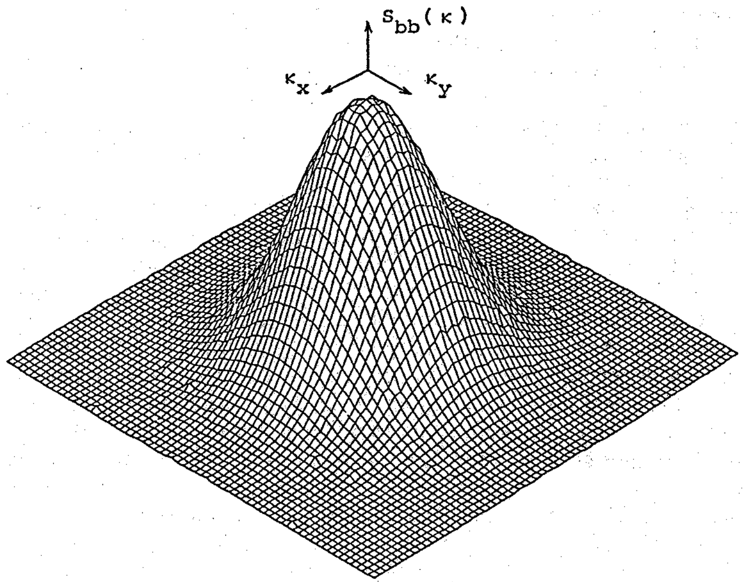

FIG. 9. Spectral Density of Simulated Beta Field

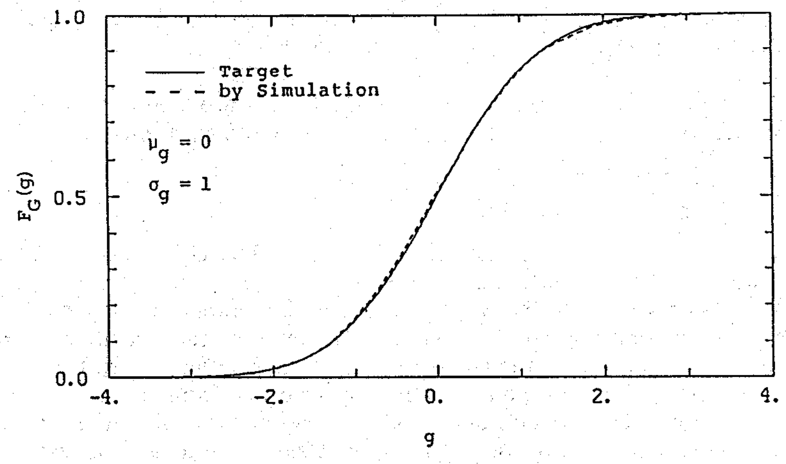

FIG. 10. Target and Simulated Gaussian Distribution Functions

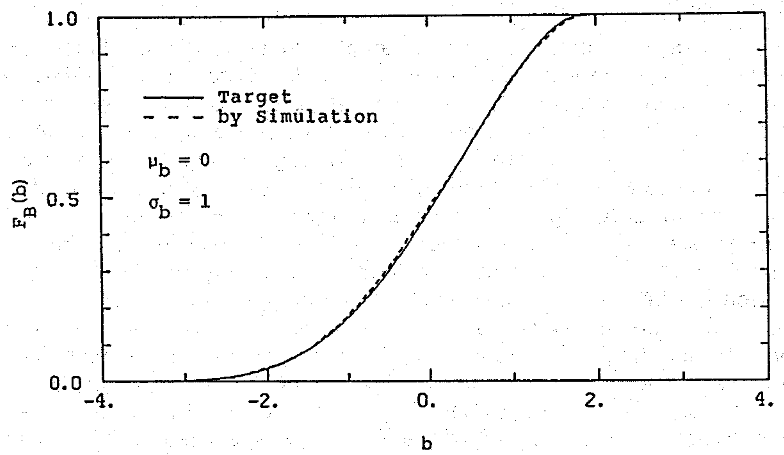

FIG. 11. Target and Simulated Beta Distribution Functions

non-Gaussian fields may also be useful especially in the solution of stochastic finite element analysis involving homogeneous but non-Gaussian fields.

COMMENTS ON INVERSION OPERATION

For any arbitrary one-dimensional probability distribution, the cumulative distribution $F_B(b)$ is a nondecreasing function with respect to b. Hence, in general, the inverse operation in Eqs. 23 and 26 can be numerically carried out by an appropriate interpolation technique. For example, an Aitken-Lagrange interpolation (see, e.g., Ralston 1965), is recommended and actually used in this study. If the first derivative of

$F_B(b)$ with respect to b (which is the probability density function $f_B[b]$) is given, an Aitken-Hermite interpolation (Gershinsky and Levine 1964) is also recommended.

The proposed algorithm is valid for any arbitrary probability distribution with the following condition: for the probability range of interest,

$$\frac{1}{m+1} \leqq F_B(b) \leqq \frac{m}{m+1} \quad \text{(37)}$$

with $m = M_x \times M_y$, and b must be a finite value. This condition is satisfied for most problems of engineering interest.

It is also noted that if the statistical distribution of the simulated field fits its target distribution, the higher-order moments at each position in the field, such as skewness and kurtosis, also fit their target values. From the engineering applications point of view, higher-order correlations involving such quantities as bispectra and trispectra may occasionally be useful. In the proposed method, however, no attempt has been made to investigate higher-order correlation characteristics. Further study in this regard is suggested.

Conclusion

A method of digitally generating sample functions of multidimensional non-Gaussian homogeneous stochastic fields is developed. First, the (target) spectral density function and (target) probability distribution function of the zero-mean non-Gaussian field are specified. Then, a corresponding Gaussian stochastic field having the same mean and variance is defined. Sample functions of the Gaussian stochastic field are generated so as to reproduce the target spectral density function of the non-Gaussian field. These sample functions are transformed into the sample functions of non-Gaussian fields by mapping on the probability distribution function diagram. Through the mapping operation, the mean and variance are unchanged. However, the spectral density of the resulting non-Gaussian field will be different from the target, since the operation is nonlinear. Then, the spectral density of the sample function of the Gaussian field is updated so that the sample function of the non-Gaussian field obtained by the mapping satisfies the target spectral density. In a few iterations of the updating of the spectral density and mapping, the sample function of the non-Gaussian field satisfies the target spectral density as well as the target probability distribution function.

A numerical example is worked out for a two-dimensional stochastic field whose one-dimensional distribution function is a beta distribution. After three iterations, the sample function closely reproduced the prescribed beta distribution and spectral density. It is noted that the proposed method can be applied to stochastic fields with any (one-dimensional) distribution form. Therefore, the method is an extremely useful tool for generating sample functions of homogeneous but non-Gaussian fields.

Acknowledgment

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Appendix. References

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